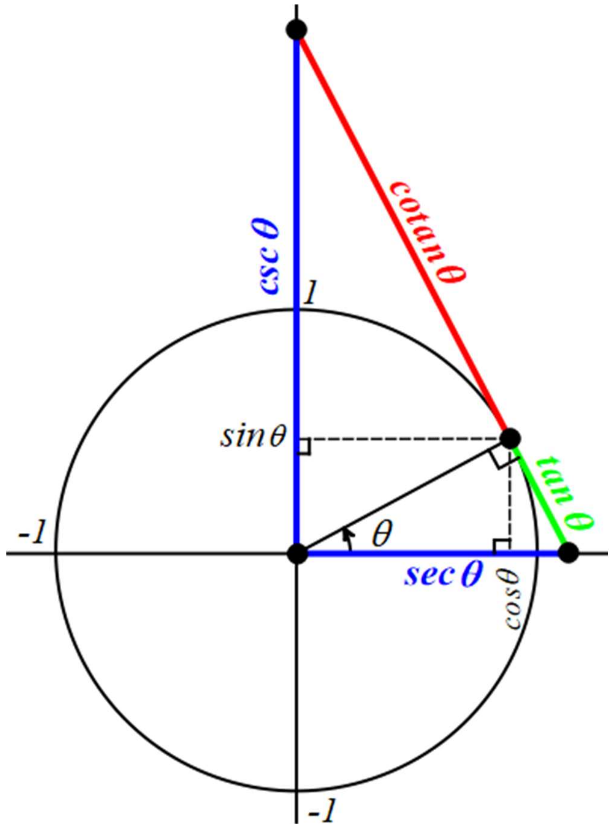
USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

Three Pythagorean identities of the unit circle:



$$sin^2\,\theta + cos^2\,\theta = 1$$

$$tan^2\,\theta + 1 = sec^2\,\theta$$

$$cotan^2\,\theta + 1 = csc^2\,\theta$$

Example 10

Prove that $\dfrac{\cos 2\theta + \sin 2\theta - 1}{\cos 2\theta - \sin 2\theta + 1} = \tan\theta$.

Solution

$$\begin{aligned}
\text{LHS} &= \frac{\cos 2\theta + \sin 2\theta - 1}{\cos 2\theta - \sin 2\theta + 1} \\
&= \frac{1 - 2\sin^2\theta + 2\sin\theta\cos\theta - 1}{2\cos^2\theta - 1 - 2\sin\theta\cos\theta + 1} \\
&= \frac{2\sin\theta(\cos\theta - \sin\theta)}{2\cos\theta(\cos\theta - \sin\theta)} \\
&= \frac{\sin\theta}{\cos\theta} \\
&= \tan\theta = \text{RHS}
\end{aligned}$$

Notice how two different expansions for $\cos 2\theta$ are used in Example 10 above. To decide which expansion is the best to use in each part you must consider the -1 in the numerator and the $+1$ in the denominator. The aim is to remove these constants by using the appropriate form.

Using $\cos 2\theta = \cos^2\theta - \sin^2\theta$ would have made the question more complicated. Try this to see for yourself.