SERIES AND SIGMA NOTATION (Σ)

1 Write expansions of the series defined by the following:

(a) $\sum_{k=1}^{4} k^2$ (b) $\sum_{r=0}^{5} 3^r$ (c) $\sum_{k=1}^{p} (2k-1)$ (d) $\sum_{k=1}^{5} k(k+1)$

a) $\sum_{k=1}^{4} k^2 = 1^2 + 2^2 + 3^2 + 4^2$

b) $\sum_{r=0}^{5} 3^r = 3^0 + 3^1 + 3^2 + 3^3 + 3^4 + 3^5$

c) $\sum_{k=1}^{p} (2k-1) = 1 + 3 + 5 + 7 + \ldots + (2p-1)$

d) $\sum_{k=1}^{5} k(k+1) = 1\times2 + 2\times3 + 3\times4 + 4\times5 + 5\times6$

(e) $\sum_{r=1}^{8} rx^r$ (f) $\sum_{r=1}^{k} \frac{1}{x^r}$ (g) $\sum_{k=1}^{p} (2k+1)^2$ (h) $\sum_{k=1}^{n+1} (3k-2)$

e) $\sum_{r=1}^{8} r x^r = 1x^1 + 2x^2 + 3x^3 + 4x^4 + 5x^5 + 6x^6 + 7x^7 + 8x^8$

f) $\sum_{r=1}^{k} \frac{1}{x^r} = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \ldots + \frac{1}{x^k}$

g) $\sum_{k=1}^{p} (2k+1)^2 = 3^2 + 5^2 + 7^2 + 9^2 + \ldots + (2p+1)^2$

h) $\sum_{k=1}^{n+1} (3k-2) = 1 + 4 + 7 + 10 + \ldots + (3n-2) + (3n+1)$

2 Indicate whether each statement below is a correct or an incorrect expression for $1 + x + x^2 + \ldots + x^{10}$.

(a) $\sum_{n=0}^{10} x^n$ (b) $\sum_{n=1}^{10} x^n$ X (c) $\sum_{n=1}^{11} x^{n-1}$ (d) $\sum_{n=1}^{10} x^{n-1}$ X

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3 Use sigma notation to represent each of the following:

(a) $1^2+2^2+3^2+\ldots+9^2$ **(b)** $1\times3+2\times4+3\times5+\ldots+10\times12$ **(c)** $1+6+11+\ldots+(5p-4)$

a) $1^2+2^2+3^2+\ldots+9^2=\sum_{k=1}^{9} k^2$

b) $1\times3+2\times4+3\times5+\ldots+10\times12=\sum_{n=1}^{10} n(n+2)$

c) $1+6+11+\ldots+(5p-4)=\sum_{n=1}^{p}(5n-4)$

(d) $\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}+\ldots+\frac{1}{p(p+1)}$ **(e)** $2x^2+3x^3+4x^4+\ldots+12x^{12}$ **(f)** $a+ar+ar^2+\ldots+ar^{n-1}$

d) $\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}+\ldots+\frac{1}{p(p+1)}=\sum_{n=2}^{p}\frac{1}{n(n+1)}$

e) $2x^2+3x^3+4x^4+\ldots+12x^{12}=\sum_{n=2}^{12} nx^n$

f) $a+ar+ar^2+\ldots+ar^{n-1}=a(1+r+r^2+\ldots+r^{n-1})$

$=a\sum_{k=0}^{n-1} r^k$

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4 Evaluate: (a) $\sum_{n=1}^{4} n^2$ (b) $\sum_{n=1}^{6} (2n+1)$ (c) $\sum_{k=1}^{4} (3k-2)$ (d) $\sum_{r=1}^{4} 2^r$

a) $\sum_{n=1}^{4} n^2 = 1^2 + 2^2 + 3^2 + 4^2$

$= 1 + 4 + 9 + 16 = 30$

b) $\sum_{n=1}^{6} (2n+1) = 3 + 5 + 7 + 9 + 11 + 13$

$= 48$

c) $\sum_{k=1}^{4} (3k-2) = 1 + (6-2) + (9-2) + (12-2)$

$= 1 + 4 + 7 + 10 = 22$

d) $\sum_{r=1}^{4} 2^r = 2^1 + 2^2 + 2^3 + 2^4$

$= 2 + 4 + 8 + 16$

$= 30$

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(e) $\sum_{n=0}^{5} n^2$ (f) $\sum_{n=0}^{5} (2n-1)$ (g) $\sum_{n=1}^{4} (n^2+n)$ (h) $\sum_{n=1}^{6} (12-3n)$ (i) $\sum_{r=1}^{4} r^r$

e) $\sum_{n=0}^{5} n^2 = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$= 0 + 1 + 4 + 9 + 16 + 25 = 55$

f) $\sum_{n=0}^{5} (2n-1) = (-1) + 1 + 3 + 5 + 7 + 9$

$= 24$

g) $\sum_{n=1}^{4} (n^2+n) = (1^2+1) + (2^2+2) + (3^2+3) + (4^2+4)$

$= 2 + 6 + 12 + 20 = 40$

h) $\sum_{n=1}^{6} (12-3n) = (12-3\times1) + (12-3\times2) + (12-3\times3)$

$+ (12-3\times4) + (12-3\times5) + (12-3\times6)$

$= 9 + 6 + 3 + 0 + (-3) + (-6)$

$= 9$

i) $\sum_{r=1}^{4} r^r = 1^1 + 2^2 + 3^3 + 4^4$

$= 1 + 4 + 27 + 256$

$= 288$