STANDARD FACTORISATIONS

Factorise:

1 $m^2 - 1$
$= (m-1)(m+1)$

2 $x^2 - 16$
$= (x-4)(x+4)$

3 $64 - m^2$
$= (8-m)(8+m)$

4 $9a^2 - 25$
$= (3a-5)(3a+5)$

5 $x^2 - 0.36$
$= (x-0.6)(x+0.6)$

6 $a^2b^2 - c^2$
$= (ab-c)(ab+c)$

7 $9x^2 - 4y^2$
$= (3x-2y)(3x+2y)$

8 $(x+1)^2 - 9$
$= (x+1-3)(x+1+3)$
$= (x-2)(x+4)$

9 $x^2 - y^2z^2$
$= (x-yz)(x+yz)$

10 $\frac{a^2}{25} - 1$
$= \left(\frac{a}{5} - 1\right)\left(\frac{a}{5} + 1\right)$

11 $p^2 - \frac{1}{4}$
$= \left(p - \frac{1}{2}\right)\left(p + \frac{1}{2}\right)$

12 $\frac{x^2}{4} - \frac{1}{9}$
$= \left(\frac{x}{2} - \frac{1}{3}\right)\left(\frac{x}{2} + \frac{1}{3}\right)$

17 $a^3b - ab^3$
$= ab(a^2 - b^2)$
$= ab(a-b)(a+b)$

18 $12a^3 - 3ab^2$
$= 3a(4a^2 - b^2)$
$= 3a(2a-b)(2a+b)$

19 $3x^2y - 27y$
$= 3y[x^2 - 9]$
$= 3y(x-3)(x+3)$

20 $(x+y)^2 - 4$
$= (x+y-2)(x+y+2)$

21 $a^2 - (a-b)^2$
$= [a - (a-b)][a + (a-b)]$
$= b(2a-b)$

22 $x^3 - x^2y - 9x + 9y$
$= x^2(x-y) - 9(x-y)$
$= (x-y)(x^2-9)$
$= (x-y)(x-3)(x+3)$

23 $x^3 + 3x^2 - 4x - 12$
$= x^2(x+3) - 4(x+3)$
$= (x+3)(x^2-4)$
$= (x+3)(x-2)(x+2)$

24 $p^2q - p^2 - 16q + 16$
$= p^2(q-1) - 16(q-1)$
$= (q-1)(p^2-16)$
$= (q-1)(p-4)(p+4)$

25 $a^2x - x$
$= x(a^2-1)$
$= x(a-1)(a+1)$

26 $48a^2 - 75b^2$
$= 3(16a^2 - 25b^2)$
$= 3(4a-5b)(4a+5b)$

27 $(1+h)^2 - 1$
$= (1+h-1)(1+h+1)$
$= h(h+2)$

28 $\frac{x^2}{25} - y^2$
$= \left(\frac{x}{5} - y\right)\left(\frac{x}{5} + y\right)$

29 When $(p+2)^2 - (p-2)^2$ is factorised, the answer is:

$= [p+2 - (p-2)][p+2 + (p-2)]$
$= 4 \times 2p$

A $2p^2 + 8$ **B** $-8p$ **C** $2p^2 - 8$ **D** $8p$

30 Indicate whether each answer is a correct or incorrect factorisation of $\frac{a^2}{b^2} - \frac{b^2}{a^2}$.

(a) $\left(\frac{a}{b} - \frac{b}{a}\right)\left(\frac{a}{b} + \frac{b}{a}\right)$ **(b)** $\left(\frac{a}{b} - \frac{b}{a}\right)\left(\frac{b}{a} + \frac{a}{b}\right)$ **(c)** $\left(\frac{a}{b} - \frac{b}{a}\right)\left(\frac{a}{b} - \frac{b}{a}\right)$ **(d)** $\frac{(a-b)(a+b)(a^2+b^2)}{a^2b^2}$

$\frac{a^2}{b^2} - \frac{b^2}{a^2} = \left(\frac{a}{b} - \frac{b}{a}\right)\left(\frac{a}{b} + \frac{b}{a}\right) = \left(\frac{a^2-b^2}{ab}\right)\left(\frac{a^2+b^2}{ab}\right)$

$= \frac{(a-b)(a+b)(a^2+b^2)}{a^2b^2}$

STANDARD FACTORISATIONS

31 $y^3 - 125$ **32** $z^3 + 1$ **33** $8p^3 + 27$ **34** $216 - a^3$

$y^3 - 125 = y^3 - 5^3 = (y-5)(y^2+5y+25)$

$z^3 + 1 = z^3 + 1^3 = (z+1)(z^2 - z + 1)$

$8p^3 + 27 = (2p)^3 + 3^3 = (2p+3)(4p^2 - 6p + 9)$

$216 - a^3 = 6^3 - a^3 = (6-a)(36 + 6a + a^2)$

A1 **35** $(x+5)^3 + (x-2)^3$ **36** $(2x+3)^3 - (x-4)^3$ **37** $b^6 - a^6$ **38** $64a^3 + 8b^3$

$(x+5)^3 + (x-2)^3 = [x+5+(x-2)][(x+5)^2 - (x+5)(x-2) + (x-2)^2]$

$= [2x+3][x^2 + x(10-3-4) + 25 + 10 + 4]$

$= (2x+3)(x^2 + 3x + 39)$

$(2x+3)^3 - (x-4)^3 = [2x+3-(x-4)][(2x+3)^2 + (2x+3)(x-4) + (x-4)^2]$

$= (x+7)[4x^2 + 2x^2 + x^2 + x(12-5-8) + 9 - 12 + 16]$

$= (x+7)[7x^2 - x + 13]$

$b^6 - a^6 = (b^3 - a^3)(b^4 + a^2b^2 + a^4)$

$= (b-a)(b+a)(b^4 + a^2b^2 + a^4)$

$64a^3 + 8b^3 = 8[8a^3 + b^3] = 8[(2a)^3 + b^3]$

$= 8[(2a+b)(4a^2 - 2ab + b^2)]$

STANDARD FACTORISATIONS

39 $\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$ 40 $p^7x^4 - p^4x^7$ 41 $x^6 + y^6$ 42 $\frac{8}{a^3} - \frac{27}{b^3}$

$\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(R^3 - r^3\right) = \frac{4}{3}\pi\left(R-r\right)\left(R^2 + rR + r^2\right)$

$p^7x^4 - p^4x^7 = p^4x^4\left(p^3 - x^3\right) = p^4x^4\left(p-x\right)\left(p^2 + xp + x^2\right)$

$x^6 + y^6 = \left(x^2\right)^3 + \left(y^2\right)^3 = \left(x^2+y^2\right)\left(x^4 - x^2y^2 + y^2\right)$

$\frac{8}{a^3} - \frac{27}{b^3} = \left(\frac{2}{a}\right)^3 - \left(\frac{3}{b}\right)^3 = \left[\frac{2}{a} - \frac{3}{b}\right]\left[\frac{4}{a^2} + \frac{6}{ab} + \frac{9}{b^2}\right]$

43 $a^3m^3 + a^3n^3 - b^3n^3 - b^3m^3$ 44 $4x^5 - 9x^3 - 4x^2 + 9$ 45 $(x+h)^3 - x^3$

$a^3m^3 + a^3n^3 - b^3n^3 - b^3m^3 = a^3\left(m^3+n^3\right) - b^3\left(n^3+m^3\right)$

$= \left(m^3+n^3\right)\left(a^3 - b^3\right)$

$= \left(m+n\right)\left(m^2 - mn + n^2\right)\left(a-b\right)\left(a^2 + ab + b^2\right)$

$4x^5 - 9x^3 - 4x^2 + 9 = x^3\left(4x^2 - 9\right) - \left(4x^2 - 9\right)$

$= \left(4x^2 - 9\right)\left(x^3 - 1\right)$

$= \left(2x-3\right)\left(2x+3\right)\left(x-1\right)\left(x^2 + x + 1\right)$

$(x+h)^3 - x^3 = \left[(x+h) - x\right]\left[(x+h)^2 + (x+h)x + x^2\right]$

$= h\left[3x^2 + x\left(2h+h\right) + h^2\right]$

$= h\left[3x^2 + 3hx + h^2\right]$

STANDARD FACTORISATIONS

46 $a^3 + (a-b)^3$ **47** $(a+b)^3 - (a-b)^3$ **48** $(2x+1)^3 - (2x-1)^3$

$a^3 + (a-b)^3 = [a + (a-b)][a^2 - a(a-b) + (a-b)^2]$

$= [2a - b][a^2 + ab - 2ab + b^2]$

$= (2a-b)(a^2 - ab + b^2)$

$(a+b)^3 - (a-b)^3 = [(a+b) - (a-b)][(a+b)^2 + (a+b)(a-b) + (a-b)^2]$

$= 2b[3a^2 + 2ab - 2ab + b^2] = 2b[3a^2 + b^2]$

$(2x+1)^3 - (2x-1)^3 = [2x+1 - (2x-1)][(2x+1)^2 + (2x+1)(2x-1) + (2x-1)^2]$

$= 2[12x^2 + 4x - 4x + 1 - 1 + 1]$

$= 2[12x^2 + 1]$

49 $8 - (2-x)^3$ **50** $a^5b^4 - a^2b$ **51** $2(x-y)^3 + 54$

$8 - (2-x)^3 = 2^3 - (2-x)^3 = [2 - (2-x)][2 + 2(2-x) + (2-x)^2]$

$= x[10 - 2x - 4x + x^2] = x[x^2 - 6x + 10]$

$a^5b^4 - a^2b = a^2b(a^3b^3 - 1) = a^2b[(ab)^3 - 1^3]$

$= a^2b[ab - 1][(ab)^2 + ab + 1]$

$2(x-y)^3 + 54 = 2[(x-y)^3 + 27] = 2[(x-y)^3 + 3^3]$

$= 2[(x-y) + 3][(x-y)^2 - 3(x-y) + 9]$

$= 2(x - y + 3)[x^2 - 2xy + y^2 - 3x + 3y + 9]$

53 When $(2x+1)^3 + (2x-1)^3$ is factorised, the answer is:

A $2(12x^2 + 1)$ **B** $4x(12x^2 + 1)$ **C** $2(4x^2 + 3)$ **D** $4x(4x^2 + 3)$

$(2x+1)^3 + (2x-1)^3 = [2x+1 + (2x-1)][(2x+1)^2 - (2x+1)(2x-1) + (2x-1)^2]$

$= 4x[4x^2 + 4x + 4x + 1 + 1 + 1]$

$= 4x(4x^2 + 3)$