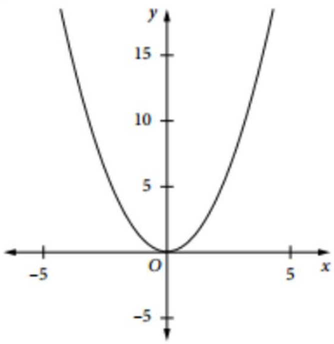
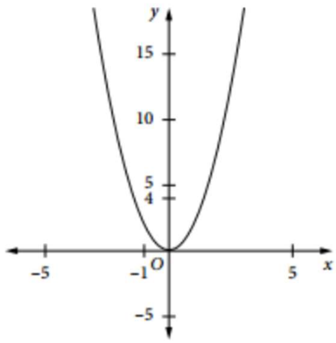
TRANSFORMATIONS OF GRAPHS USING $y = f(ax)$ AND $y = f(a(x+b))$

Consider the following graphs.

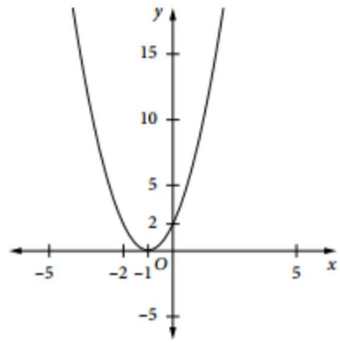
$y = x^2$



$y = (2x)^2$



$y = (2(x+1))^2$



If $y = x^2$ is written as $y = f(x)$ then $y = (2x)^2$ becomes $y = f(2x)$ and $y = (2(x+1))^2$ becomes $y = f(2(x+1))$.
In $y = f(2x)$ the curve for $y = f(x)$ has stretched by a factor of $\frac{1}{2}$ unit from the y-axis.
In $y = f(2(x+1))$ the curve for $y = f(x)$ has been moved 1 unit to the left and then been stretched by a factor of $\frac{1}{2}$ unit from the y-axis.

Summary – Transformations of graphs

Given $y = f(x)$, then:
- $y = f(x) + c$ translates the curve c units up
- $y = f(x+b)$ translates the curve b units to the left
- $y = kf(x+b)$ stretches (dilates) the curve by a factor of k from the x-axis
- $y = f(a(x+b))$ stretches (dilates) the curve by a factor $\frac{1}{a}$ from the y-axis.

Example 2

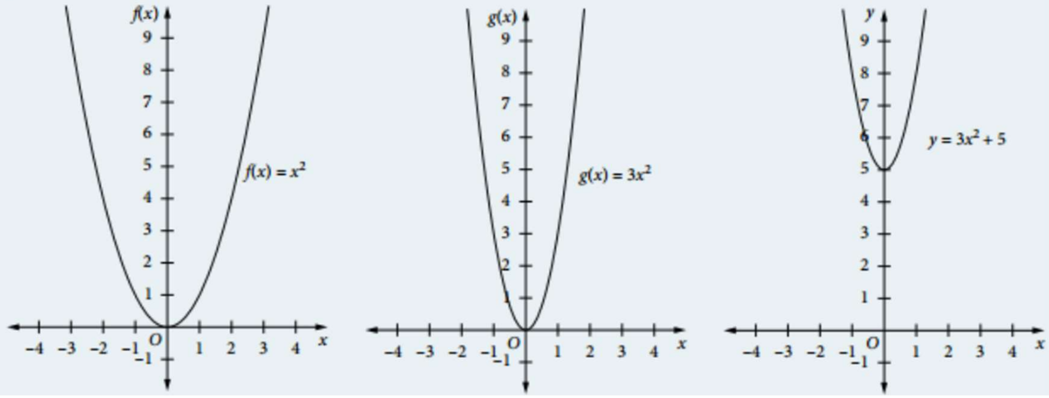
(a) On successive diagrams, draw the graphs of $f(x) = x^2$, $g(x) = 3f(x)$ and $y = g(x) + 5$.

(b) On successive diagrams, draw the graphs of $f(x) = x^2$, $g(x) = f(x) + 5$ and $y = 3g(x)$.

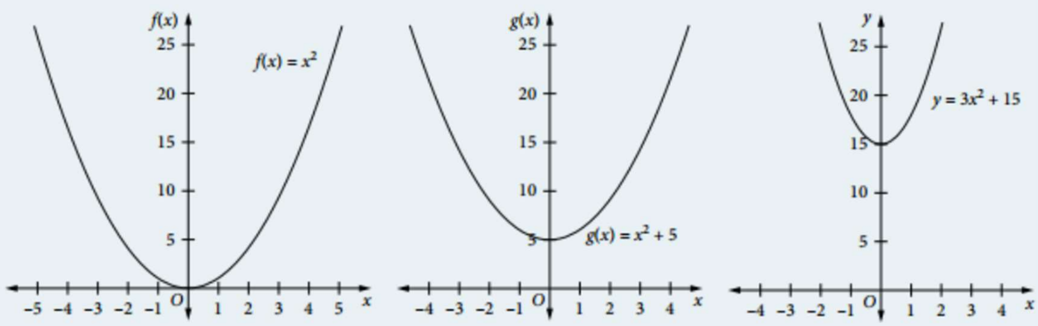
(c) Discuss the differences between your final graphs in parts **(a)** and **(b)**.

Solution

(a)



(b)



(c) In part **(b)** the vertical translation was 15 instead of 5. The dilation was the same in each case.

The order in which a series of transformations is applied to a function is important. Reversing or changing the order can change the final function.