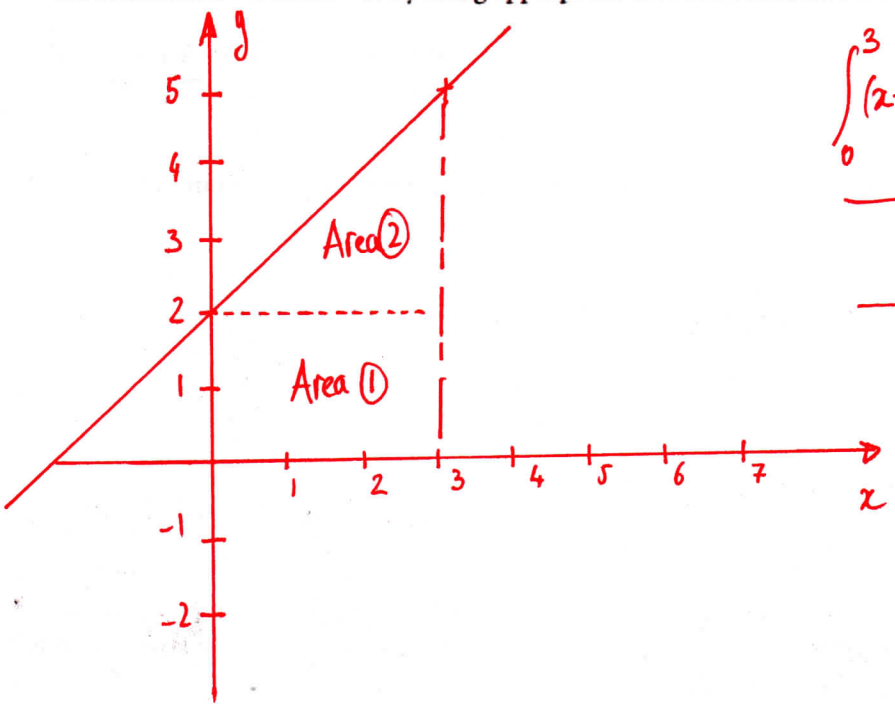
# THE DEFINITE INTEGRAL AND THE AREA UNDER A CURVE

**1** Write the definite integral that you would use to find the area of the region under the line $y = x + 2$ between the ordinates $x = 0$ and $x = 3$. By using appropriate area formulae, find the value of this area.




$\int_0^3 (x+2)\,dx$ = Area ① + Area ②

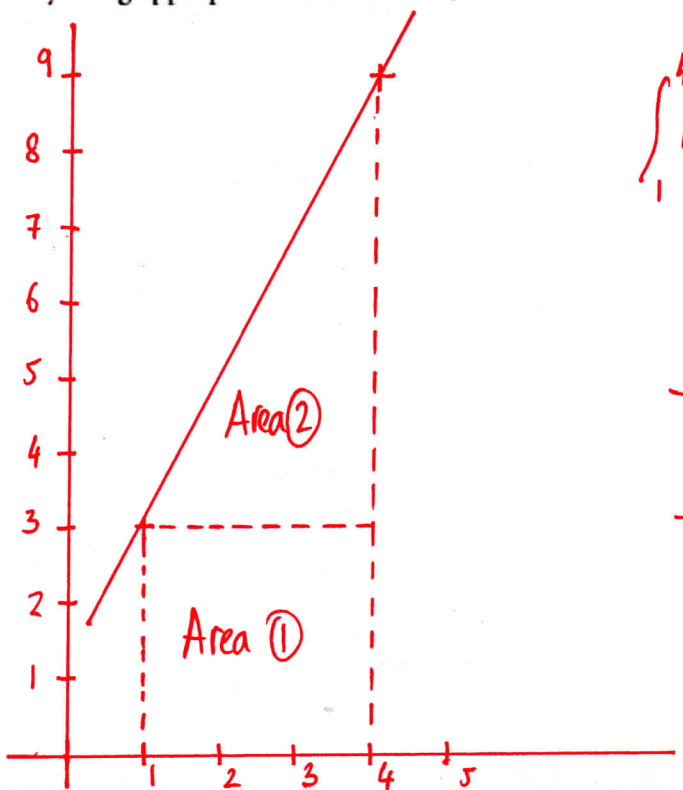
$= 3 \times 2 + \frac{1}{2} \times 3 \times 3$

$= 6 + \frac{9}{2}$

$= 10.5$ square units

**2** Write the definite integral for the area of the region under the line $y = 2x + 1$ between $x = 1$ and $x = 4$. By using appropriate area formulae, find the value of this area.

$x = 1 \quad y = 3 \qquad x = 4 \quad y = 9$




$\int_1^4 (2x+1)\,dx$ = Area ① + Area ②

$= 3 \times 3 + \frac{3 \times 6}{2}$

$= 9 + 9$

$= 18$ square units

**3** Which definite integral represents the area bounded by the curve $y = 4 - x^2$ and the $x$-axis?

**A** $\int_0^2 (4 - x^2)\,dx$ **B** $\int_{-2}^0 (4 - x^2)\,dx$ **C** $\int_{-2}^2 (4 - x^2)\,dx$ **D** $\int_{-\sqrt{2}}^{\sqrt{2}} (4 - x^2)\,dx$

$y = 0$ when $x = \pm 2$ so C

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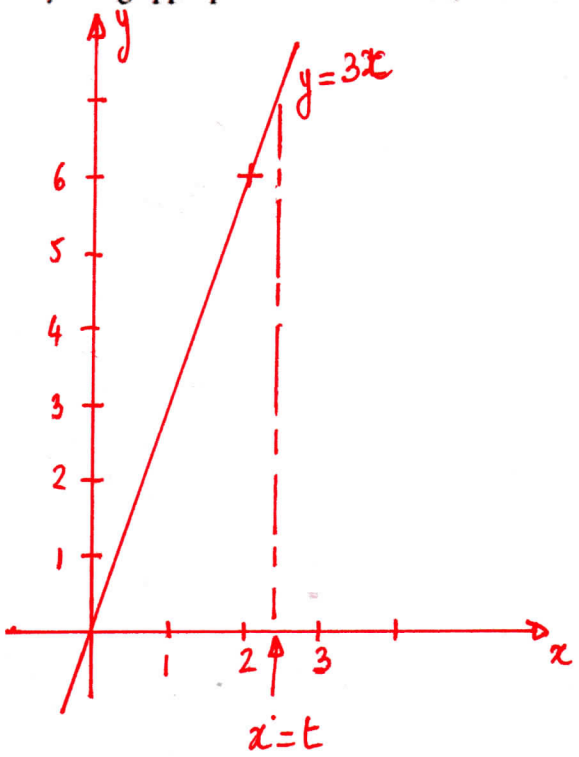
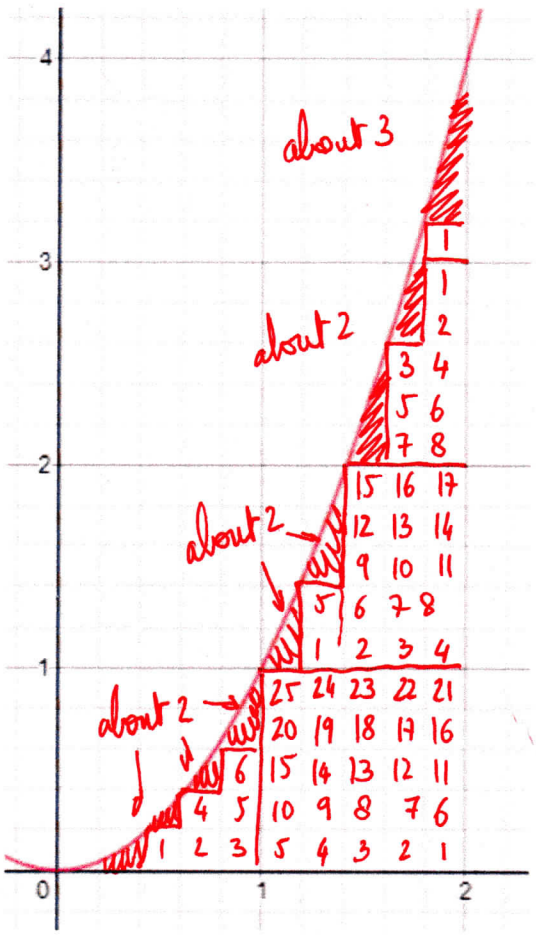
**4** Write the definite integral for the area of the region bounded by the lines $y = 3x$, $x = t$ and the $x$-axis. By using appropriate area formulae, find the value of this area.




$$\int_0^t 3x\,dx = \frac{t \times 3t}{2}$$

$$= \frac{3t^2}{2} \text{ square units.}$$

**5** Write the definite integral for the area of the region under the curve $y = x^2$ between $x = 0$ and $x = 2$. By drawing this graph on a 5 mm grid, count squares to find an approximation for this area.




$$\int_0^2 x^2\,dx \approx 66 \times \frac{1}{5^2}$$

$$\approx 2.64 \text{ square units}$$

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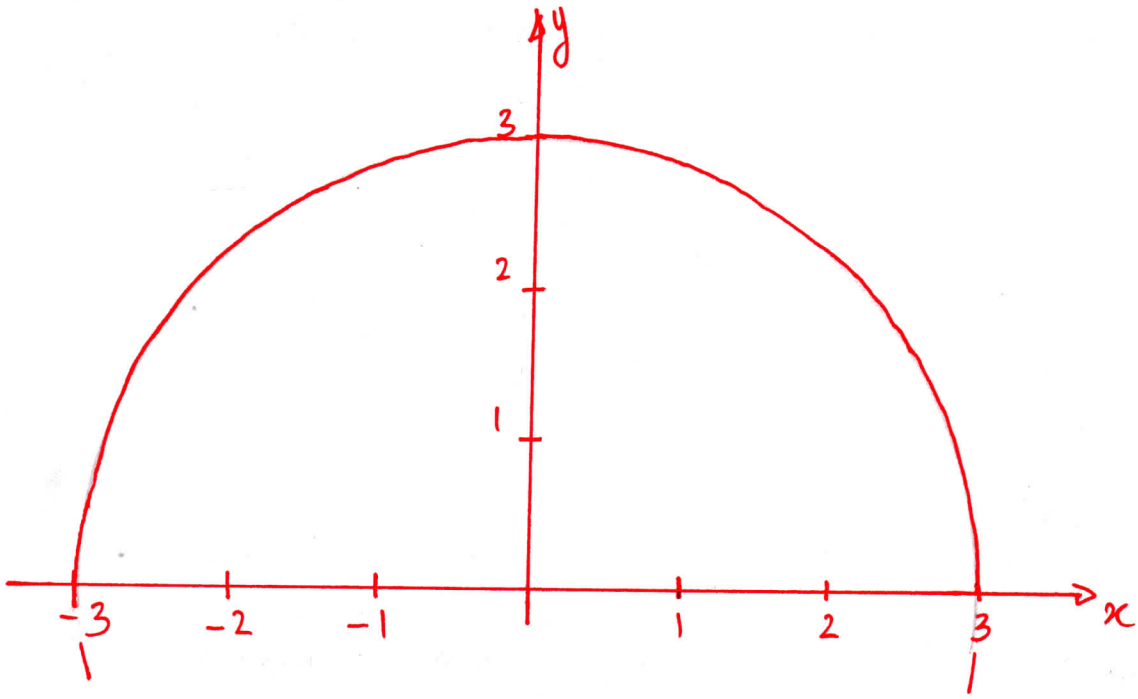
**7** Write the definite integral for the area of the region under the curve $y = \sqrt{9 - x^2}$. By using appropriate area formulae, find the value of this area.

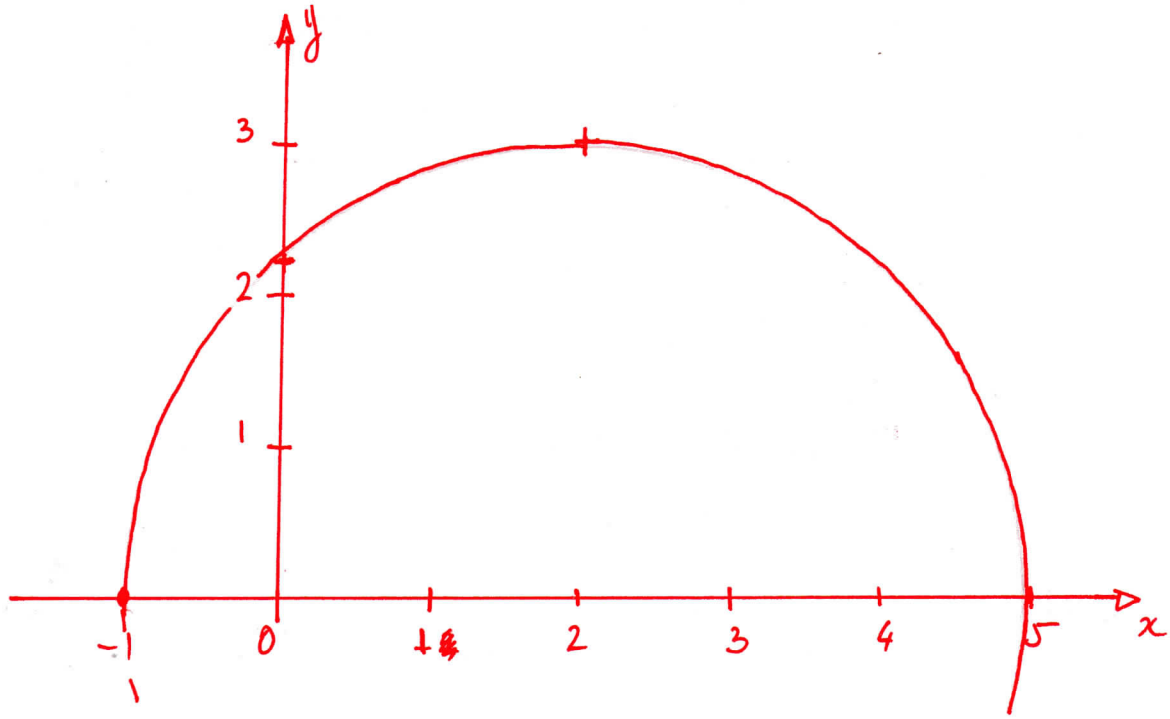
$y = 0$ when $x = \pm 3$




$$\therefore \int_{-3}^{3} \sqrt{9 - x^2}\, dx = \frac{1}{2} \pi \times 3^2 = \frac{9\pi}{2}$$

# THE DEFINITE INTEGRAL AND THE AREA UNDER A CURVE

**8** Write the definite integral for the area of the region under the curve $y = \sqrt{9-(x-2)^2}$. By using appropriate area formulae, find the value of this area.




$$\int_{-1}^{5} \sqrt{9-(x-2)^2}\,dx = \frac{1}{2}\pi 3^2 = \frac{9\pi}{2}$$