TRIGONOMETRIC IDENTITIES AND PROOFS

1 If $\tan\theta = \frac{7}{24}$ and $\pi < \theta < \frac{3\pi}{2}$, find the exact value of:

(a) $\sin\theta$ **(b)** $\cos\theta$

2 If $\sin\theta = \frac{5}{13}$ and θ is acute, indicate whether each statement is correct or incorrect.

(a) $\cos\theta = \frac{12}{13}$ **(b)** $\sec\theta = \frac{13}{5}$ **(c)** $\tan\theta = \frac{5}{12}$ **(d)** $\cot\theta = \frac{13}{12}$

4 If $\cos u = \frac{2}{3}$ and u is not in the first quadrant, then $\frac{\cos u - 2\cot u}{\tan u - 3\sin u}$ simplifies to:

A $\frac{4\left(\sqrt{5}+6\right)}{15}$ **B** $\frac{5-2\sqrt{5}}{9}$ **C** $\frac{2\left(5-\sqrt{5}\right)}{21}$ **D** $\frac{14}{3\left(4\sqrt{5}-5\right)}$

5 Simplify:

(a) $\dfrac{\sin^2\theta+\cos^2\theta}{\tan^2\theta}$ (b) $\dfrac{\sin^2\theta}{1-\sin^2\theta}$ (c) $\dfrac{2\cot\alpha}{1+\cot^2\alpha}$ (d) $(\sec^2\theta-1)\tan(\dfrac{\pi}{2}-\theta)$

5 Simplify:

(e) $\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}$ **(f)** $\sin^3\theta+\sin\theta\cos^2\theta$ **(g)** $\operatorname{cosec}^2\theta\sin^2\theta$ **(h)** $1-\sin^2(\pi+\theta)$

6 Simplify:

(a) $\dfrac{x^2}{\sqrt{x^2 - a^2}}$ for $x = a\sec\theta$ **(b)** $\sqrt{a^2 - x^2}$ for $x = a\cos\theta$

9 Find the exact value of $\sec\theta$ if $\tan\theta = 0.6$ and θ is not in the first quadrant.

10 If $\sin\theta = x$, express $\dfrac{1-\cos^2\theta}{\sec^2\theta}$ in terms of x.

11 If $a\sin^2\theta + b\cos^2\theta = c$, express $\sin\theta$ and $\cos\theta$ in terms of a, b and c.

13 If $\tan^2\theta + 2\sec^2\theta = 5$, find the value of $\sin^2\theta$.

14 Simplify:

(a) $(1+\tan^2 u)(1-\sin^2 u)$ **(b)** $\frac{1}{1-\sin V}+\frac{1}{1+\sin V}$ **(g)** $2\cos^2\frac{\pi}{6}-1$ **(h)** $1-\sin\theta\cos(\frac{\pi}{2}-\theta)$

Prove the following identities.

15 $(1-\tan x)^2+(1+\tan x)^2=2\sec^2 x$

16 $(\cot t+\operatorname{cosec} t)^2=\dfrac{1+\cos t}{1-\cos t}$

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Prove the following identities.

17 $\sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta = \sin^2\alpha - \sin^2\beta$

18 $\sec\theta + \tan\theta = \dfrac{1+\sin\theta}{\cos\theta}$

TRIGONOMETRIC IDENTITIES AND PROOFS

Prove the following identities.

21 $\tan\theta(1-\cot^2\theta)+\cot\theta(1-\tan^2\theta)=0$

22 $\dfrac{\cot\theta\cos\theta}{\cot\theta+\cos\theta}=\dfrac{\cos\theta}{1+\sin\theta}$