QUADRATIC EQUATIONS WITH NON-RATIONAL SOLUTIONS

All the questions in Exercise 3.9 could have been solved by factorising. Practice in completing the square will also enable you to solve equations that do not have rational factors.

Example 14

Complete the square to solve:

(a) $x^2 + 2x - 5 = 0$ **(b)** $x^2 = 4x + 8$ **(c)** $x^2 - 5x + 2 = 0$

Solution

(a) $x^2 + 2x - 5 = 0$

$x^2 + 2x = 5$	Move constant to RHS
$x^2 + 2x + 1 = 5 + 1$	Add 1^2 to complete the square
$(x+1)^2 = 6$	Factorise
$x + 1 = \pm\sqrt{6}$	Take square roots of both sides
$x = -1 + \sqrt{6}$ or $x = -1 - \sqrt{6}$	Exact answers
$x = 1.45$ or $x = -3.45$	Answers correct to 2 decimal places

(b) $x^2 = 4x + 8$

$x^2 - 4x = 8$	Rewrite with only the constant on RHS
$x^2 - 4x + 4 = 8 + 4$	Add 2^2 to complete the square
$(x-2)^2 = 12$	Factorise
$x - 2 = \pm\sqrt{12}$	Take square roots of both sides
$x = 2 + 2\sqrt{3}$ or $x = 2 - 2\sqrt{3}$	Exact answers
$x = 5.46$ or $x = -1.46$	Answers correct to 2 decimal places

(c) $x^2 - 5x + 2 = 0$

$x^2 - 5x = -2$	Move constant to RHS
$x^2 - 5x + \frac{25}{4} = -2 + \frac{25}{4}$	Add $\left(\frac{5}{2}\right)^2$ to complete the square
$\left(x - \frac{5}{2}\right)^2 = \frac{17}{4}$	Factorise
$x - \frac{5}{2} = \pm\frac{\sqrt{17}}{2}$	Take square roots of both sides
$x = \frac{5}{2} + \frac{\sqrt{17}}{2}$ or $x = \frac{5}{2} - \frac{\sqrt{17}}{2}$	Exact answers
$x = 4.56$ or $x = 0.44$	Answers correct to 2 decimal places