PRIMITIVE FUNCTIONS

1 Find the primitive of:

(a) $6x^2 - 4x + 5$ **(b)** $3 + 5x + x^2 - 3x^3$ **(c)** $x^2 - 1$

a) $\int (6x^2 - 4x + 5)\,dx = 6\frac{x^3}{3} - 4\frac{x^2}{2} + 5x + C = 2x^3 - 2x^2 + 5x + C$

b) $\int (3 + 5x + x^2 - 3x^3)\,dx = 3x + 5\frac{x^2}{2} + \frac{x^3}{3} - 3\frac{x^4}{4} + C$

$= 3x + \frac{5x^2}{2} + \frac{x^3}{3} - \frac{3x^4}{4} + C$

c) $\int (x^2 - 1)\,dx = \frac{x^3}{3} - x + C$

2 If $f'(x) = (x-1)(x-2)$, indicate whether each statement below is correct or incorrect.

(a) $f'(x) = x^2 - 3x + 2$ **(b)** $f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + C$

(c) $f(x) = (x-1)^2(x-2)^2 + C$ **(d)** $f(x) = \frac{(x-1)^2(x-2)^2}{4} + C$

$f'(x) = x^2 - x - 2x + 2 = x^2 - 3x + 2$ so a) correct

$f(x) = \frac{x^3}{3} - 3\frac{x^2}{2} + 2x + C$ so b) correct

c) incorrect as $(x-1)^2(x-2)^2$ has degree 4

So as d) also incorrect

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3 Find an expression for $f(x)$ given:

(a) $f'(x) = (2x+1)^2$ (b) $f'(x) = 5$ (c) $f'(x) = x^2 + 3x$

a) $\int (2x+1)^2 dx = \int (4x^2 + 4x + 1)\, dx = \frac{4x^3}{3} + 4\frac{x^2}{2} + x + C$

$= \frac{4x^3}{3} + 2x^2 + x + C$

b) $\int 5\, dx = 5x + C$ so $f(x) = 5x + C$

c) $\int (x^2 + 3x)\, dx = \frac{x^3}{3} + \frac{3x^2}{2} + C$

so $f(x) = \frac{x^3}{3} + \frac{3x^2}{2} + C$

5 Express y in terms of x, given that:

(a) $\frac{dy}{dx} = 3 + 2x - 3x^2$ (b) $\frac{dy}{dx} = x^3 + 2x^2$ (c) $\frac{dy}{dx} = x^4 - x^3$

a) $y = \int (3 + 2x - 3x^2)\, dx = 3x + 2\frac{x^2}{2} - 3\frac{x^3}{3} + C = 3x + x^2 - x^3 + C$

b) $y = \int (x^3 + 2x^2)\, dx = \frac{x^4}{4} + 2\frac{x^3}{3} + C$

c) $y = \int (x^4 - x^3)\, dx = \frac{x^5}{5} - \frac{x^4}{4} + C$

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7 Find $f(x)$ given $f'(x) = 2x - 2$ and $f(1) = 4$.

$f(x) = \int (2x - 2)\,dx = \frac{2x^2}{2} - 2x + C = x^2 - 2x + C$

But $f(1) = 4$ so $1^2 - 2 \times 1 + C = 4$

or $-1 + C = 4$ so $C = 5$

$f(x) = x^2 - 2x + 5$

9 At all points on a certain curve, $\frac{dy}{dx} = 4x - 6$. The point $(2, 4)$ is on the curve. Find the equation of the curve.

$f(x) = \int (4x - 6)\,dx = 4\frac{x^2}{2} - 6x + C = 2x^2 - 6x + C$

But $f(2) = 4$ so $4 = 2 \times 2^2 - 6 \times 2 + C$

$4 = 8 - 12 + C$

$4 = -4 + C$

So $C = 8$ $\quad f(x) = 2x^2 - 6x + 8$

10 Find the equation of a curve that passes through the point $(3, 3)$ and for which the gradient function at any point $P(x, y)$ is $3x^2 - 2x + 3$.

$f(x) = \int (3x^2 - 2x + 3)\,dx = 3\frac{x^3}{3} - 2\frac{x^2}{2} + 3x + C$

So $f(x) = x^3 - x^2 + 3x + C$

But $f(3) = 3$ so $3 = 3^3 - 3^2 + 3 \times 3 + C$

$3 = 27 - 9 + 9 + C$

$3 = 27 + C$ so $\boxed{C = -24}$

$f(x) = x^3 - x^2 + 3x - 24$

12 Find the equation of a curve given that $\frac{dy}{dx} = 2x + b$ at any point P and that when $x = 3$, $\frac{dy}{dx} = 2$ and $y = -3$.

$$y = \int (2x + b)\,dx = 2\frac{x^2}{2} + bx + C = x^2 + bx + C$$

But $\frac{dy}{dx} = 2$ when $x = 3$ so $2 \times 3 + b = 2$

so $6 + b = 2$ $\boxed{b = -4}$

and $y = -3$ when $x = 3$

so $-3 = 3^2 - 4 \times 3 + C$

$-3 = 9 - 12 + C$ so $-3 = -3 + C$

so $C = 0$ $\boxed{y = x^2 - 4x}$

18 If velocity v is the rate of change of distance d as a function of time t, find the distance function if $v = 3t^2 + 4$ and $d = 0$ when $t = 0$.

$V = \frac{d(d)}{dt}$ So $d(t) = \int (3t^2 + 4)\,dt$

$$d(t) = 3\frac{t^3}{3} + 4t + C = t^3 + 4t + C$$

at $t = 0$, $d = 0$ so $0 = 0^3 + 4 \times 0 + C$

so $C = 0$

$$\boxed{d(t) = t^3 + 4t}$$