

Example 4

(a) Solve the equation $\tan^2 \theta + \tan \theta - 2 = 0$, $0^\circ < \theta < 360^\circ$.

(b) Solve the equation $\cos^2 x = 2 \cos x$, $-\pi \leq x \leq \pi$.

Solution

(a) Factorise: $(\tan \theta - 1)(\tan \theta + 2) = 0$

$$\tan \theta = 1 \quad \text{or} \quad -2$$

$$\theta = 45^\circ, 225^\circ \quad \text{or} \quad 116^\circ 34', 296^\circ 34'$$

Solution is $\theta = 45^\circ, 116^\circ 34', 225^\circ, 296^\circ 34'$.

(b) Rearrange: $\cos^2 x - 2 \cos x = 0$

$$\text{Factorise: } \cos x (\cos x - 2) = 0$$

$$\therefore \cos x = 0 \quad \text{or} \quad 2$$

Because $|\cos x| \leq 1$, the only solution is $\cos x = 0$.

$$\therefore x = -\frac{\pi}{2}, \frac{\pi}{2}$$

Example 5

Solve the equation $\sec^2 x - 2 \tan x = 4$ for $0 \leq x \leq 2\pi$.

(Trigonometric values rounded to 3 d.p. where necessary.)

Solution

The trigonometric functions are different, but they can be linked by the identity $\sec^2 x = 1 + \tan^2 x$:

$$\sec^2 x - 2 \tan x = 4$$

$$1 + \tan^2 x - 2 \tan x = 4$$

$$\tan^2 x - 2 \tan x - 3 = 0$$

$$(\tan x - 3)(\tan x + 1) = 0$$

$$\tan x = -1 \quad \text{or} \quad 3$$

$$x = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}, 1.249, \pi + 1.249$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}, 1.249, 4.391$$