QUESTION 1 Complete:

a If $\log_a x = c$ then $x =$ a^c

b $\log_a xy = \log_a x +$ $\log_a y$

c $\log_a \frac{x}{y} =$ $\log_a x - \log_a y$

d $\log_a a =$ 1

e $\log_a 1 =$ 0

f $\log_a x^n =$ $n \times \log_a x$

QUESTION 2 Express as an integer:

a $\log_3 27 = 3$ as $3^3 = 27$

b $\log_2 32 = 5$ as $2^5 = 32$

c $\log_5 5 = 1$ as $5^1 = 5$

d $\log_7 1 = 0$ as $7^0 = 1$

e $\log_6 36 = 2$ as $6^2 = 36$

f $\log_{10} 100\,000 = 5$ as $10^5 = 100{,}000$

g $\log_7 343 = 3$ as $7^3 = 343$

h $\log_2 256 = 8$ as $2^8 = 256$

1 Write each expression in logarithmic form.

a $9 = 3^2$ $\log_3 9 = 2$

b $8 = 2^3$ $\log_2 8 = 3$

c $6^2 = 36$ $\log_6 36 = 2$

d $2^5 = 32$ $\log_2 32 = 5$

e $8^1 = 8$ $\log_8 8 = 1$

f $5^0 = 1$ $\log_5 1 = 0$

g $64 = 4^3$ $\log_4 64 = 3$

h $3^5 = 243$ $\log_3 243 = 5$

i $5^{-1} = \frac{1}{5}$ $\log_5 \left(\frac{1}{5}\right) = -1$

j $2^{-2} = \frac{1}{4}$ $\log_2 \left(\frac{1}{4}\right) = -2$

k $\sqrt{3} = 3^{\frac{1}{2}}$ $\log_3 \sqrt{3} = \frac{1}{2}$

l $\sqrt[3]{7} = 7^{\frac{1}{3}}$ $\log_7 \left(\sqrt[3]{7}\right) = \frac{1}{3}$

m $9^{\frac{3}{2}} = 27$ $\log_9 27 = \frac{3}{2}$

n $\sqrt[5]{8} = 2^{\frac{3}{5}}$ $\log_2 \sqrt[5]{8} = \frac{3}{5}$

o $144^{-\frac{1}{2}} = \frac{1}{12}$ $\log_{144} \left(\frac{1}{12}\right) = -\frac{1}{2}$

p $16^{-\frac{3}{4}} = \frac{1}{8}$ $\log_{16} \left(\frac{1}{8}\right) = -\frac{3}{4}$

2 Write each expression in index form.

a $\log_5 25 = 2$ $5^2 = 25$

b $\log_2 16 = 4$ $2^4 = 16$

c $\log_7 7 = 1$ $7^1 = 7$

d $\log_2 8 = 3$ $2^3 = 8$

e $\log_3 81 = 4$ $3^4 = 81$

f $\log_{10} 100 = 2$ $10^2 = 100$

g $\log_2 32 = 5$ $2^5 = 32$

h $\log_6 1 = 0$ $6^0 = 1$

i $\log_3 243 = 5$ **j** $\log_2\left(\frac{1}{2}\right) = -1$ **k** $\log_3\left(\frac{1}{9}\right) = -2$ **l** $\log_4 2 = \frac{1}{2}$

$3^5 = 243$ $2^{-1} = \frac{1}{2}$ $3^{-2} = 1/9$ $4^{1/2} = 2$

m $\log_{27} 3 = \frac{1}{3}$ **n** $\log_5 \sqrt{5} = \frac{1}{2}$ **o** $\log_8 4 = \frac{2}{3}$ **p** $\log_{1000}\left(\frac{1}{100}\right) = -\frac{2}{3}$

$27^{1/3} = 3$ $5^{1/2} = \sqrt{5}$ $8^{2/3} = 4$ $1000^{(-2/3)} = \frac{1}{100}$

3 Evaluate:

a $\log_{10} 10$ **b** $\log_4 16$ **c** $\log_3 27$ **d** $\log_2 32$

$= 1$ $= 2$ $= 3$ $= 5$

e $\log_9 1$ **f** $\log_{10} 1000$ **g** $\log_6 36$ **h** $\log_3 81$

$= 0$ $= 3$ $= 2$ $= 4$

i $\log_2 \sqrt{2}$ **j** $\log_8 2$ **k** $\log_4\left(\frac{1}{4}\right)$ **l** $\log_3\left(\frac{1}{9}\right)$

$= 1/2$ $= 1/3$ $= -1$ $= -2$

5 Solve for x.

a $\log_x 8 = 3$ **b** $\log_x 9 = 2$ **c** $\log_x 13 = 1$ **d** $\log_x 64 = 6$

$x = 2$ $x = 3$ $x = 13$ $x = 2$

e $\log_x 121 = 2$ **f** $\log_x 64 = 3$ **g** $\log_x 243 = 5$ **h** $\log_x 10\,000 = 4$

$x = 11$ $x = 4$ $x = 3$ $x = 10$

i $\log_x\left(\frac{1}{5}\right) = -1$ **j** $\log_x\left(\frac{1}{36}\right) = -2$ **k** $\log_x\left(\frac{1}{16}\right) = -4$ **l** $\log_x \sqrt{6} = \frac{1}{2}$

$x = 5$ $x = 6$ $x = 2$ $x = 6$

m $\log_x 2 = \frac{1}{3}$ **n** $\log_x 4 = \frac{2}{3}$ **o** $\log_x 27 = \frac{3}{2}$ **p** $\log_x 64 = \frac{6}{5}$

$x = 8$ $x^{2/3} = 4$ $x^{3/2} = 27$ $x^{6/5} = 64$

as $8^{1/3} = \sqrt[3]{8} = 2$ so $x^2 = 4^3 = 64$ so $x^3 = 27^2 = 729$ $x^6 = 64^5$

so $x = 8$ $x = \sqrt[3]{729} = 9$ $x = \sqrt[6]{64^5} = 32$

2 Write the missing numbers.

a $\log_{\boxed{2}} = 1$ **b** $\log_{\boxed{1}} = 0$ **c** $\log_{\boxed{2}} 32 = 5$

d $\log_a 2 + \log_a \boxed{4} = \log_a 8$ **e** $\log_a 36 - \log_a \boxed{12} = \log_a 3$ **f** $\boxed{3}\log_a 5 = \log_a 5^3$

g $\log_a 3^4 = \boxed{4}\log_a 3$ **h** $\log_a \frac{1}{7} = \log_a 7^{\boxed{-1}}$ **i** $-\boxed{1}\log_a 3 = \log_a \frac{1}{3}$

4 Simplify:

a $\log_a 3 + \log_a 2$

$= \log_a 6$

b $\log_a 5 + \log_a 3$

$= \log_a 15$

c $\log_a 7 + \log_a 4$

$= \log_a 28$

d $\log_b 6 + \log_b 3$

$= \log_b 18$

e $\log_b 15 + \log_b 1$

$= \log_b 15$

f $\log_b 1 + \log_b 17$

$= \log_b 17$

a $\log_a 10 - \log_a 5$

$= \log_a \left(\frac{10}{5}\right) = \log_a 2$

b $\log_a 36 - \log_a 12$

$= \log_a \left(\frac{36}{12}\right) = \log_a 3$

c $\log_a 100 - \log_a 10$

$= \log_a \left(\frac{100}{10}\right) = \log_a 10$

d $\log_b 28 - \log_b 14$

$= \log_b \left(\frac{28}{14}\right) = \log_b 2$

e $\log_b 3 - \log_b 2$

$= \log_b \left(\frac{3}{2}\right)$

f $\log_b 7 - \log_b 5$

$= \log_b \left(\frac{7}{5}\right)$

1 Evaluate each of the following using the logarithm laws.

a $\log_{12} 3 + \log_{12} 4$

$= \log_{12}(3 \times 4) = \log_{12} 12 = 1$

b $\log_3 15 - \log_3 5$

$= \log_3 \left(\frac{15}{5}\right) = \log_3 3 = 1$

c $\log_6 12 + \log_6 3$

$= \log_6 (12 \times 3) = \log_6 36 = 2$

d $\log_2 48 - \log_2 6$

$= \log_2 \left(\frac{48}{6}\right) = \log_2 8 = 3$

e $\log_2 80 - \log_2 5$

$= \log_2 \left(\frac{80}{5}\right) = \log_2 16 = 4$

f $\log_7 2 + \log_7 \left(\frac{1}{2}\right)$

$= \log_7 \left(2 \times \frac{1}{2}\right) = \log_7 1 = 0$

g $\log_4 36 - 2\log_4 3$

$= \log_4 36 - \log_4 3^2$

$= \log_4\left(\frac{36}{9}\right) = \log_4 4 = 1$

h $3\log_{10} 5 + \log_{10} 8$

$= \log_{10} 5^3 + \log_{10} 8$

$= \log_{10} 1000 = 3$

i $\log_{100} 40 - \log_{100} 4$

$= \log_{100}\left(\frac{40}{4}\right) = \log_{100} 10 = \frac{1}{2}$

j $\log_2 5 - \log_2 40$

$= \log_2\left(\frac{5}{40}\right) = \log_2\left(\frac{1}{8}\right)$

$= \log_2\left(\frac{1}{2^3}\right) = -3$

k $4\log_8 2 + \frac{1}{2}\log_8 16$

$= \log_8 2^4 + \log_8 16^{1/2}$

$= \log_8 2^4 \times 16^{1/2}$

$= \log_8 64 = 2.$

l $2\log_{10} 5 - \log_{10}\left(\frac{1}{4}\right)$

$= \log_{10} 5^2 - \log_{10}\left(\frac{1}{4}\right)$

$= \log_{10}\left(\frac{25}{\frac{1}{4}}\right) = \log_{10} 100 = 2$

9 If $\log_a 5 = 1.16$ and $\log_a 10 = 1.66$, evaluate:

a $\log_a 5a$

$= \log_a 5 + \log_a a$

$= 1.16 + 1 = 2.16$

b $\log_a\left(\frac{10}{a}\right)$

$= \log_a 10 - \log_a a$

$= 1.66 - 1 = 0.66$

c $\log_a\left(\frac{a}{5}\right)$

$= \log_a a - \log_a 5$

$= 1 - 1.16 = -0.16$

d $\log_a 25a$

$= \log_a 25 + \log_a a$

$= 2\log_a 5 + 1 = 2 \times 1.16 + 1$

$= 3.32$

e $\log_a 10a^2$

$= \log_a 10 + \log_a a^2$

$= 1.66 + 2\times\log_a a$

$= 1.66 + 2 = 3.66$

f $\log_a 125a^3$

$= \log_a 5^3 + \log_a a^3$

$= 3 \times 1.16 + 3 \times 1 = 6.48$

g $\log_a a\sqrt{5}$

$= \log_a a + \log_a 5^{1/2}$

$= 1 + \frac{1}{2} \times 1.16 = 1.58$

h $\log_a \sqrt{5a}$

$= \log_a (5a)^{1/2} = \frac{1}{2}\log_a 5a.$

$= \frac{1}{2}\left[\log_a 5 + \log_a a\right] = \frac{1}{2}\left[1.16 + 1\right]$

$= 1.08$

i $\log_a\left(\frac{a^2}{10}\right)$

$= \log_a a^2 - \log_a 10$

$= 2\log_a a - 1.66 = 2 \times 1 - 1.66$

$= 0.34$

j $\log_a 2a$

$= \log_a\left(\frac{10a}{5}\right) = \log_a(10a) - \log_a 5$

$= 1.66 - 1 - 1.16 = 1.5$

k $\log_a\left(\frac{a}{2}\right)$

$= \log_a a - \log_a 2$

$= 1 - \log_a\frac{10}{5} = 1 - (1.66 - 1.16)$

$= 0.5$

l $\log_a\left(\frac{1}{2a}\right)$

$= \log_a 1 - \log_a 2a$

$= -\left[\log_a 2 - \log_a a\right]$

$= 1 - \log_a\frac{10}{5} = 1 - (1.66 - 1.16)$

$= -1.5$

6 Solve $\log_5[\log_5(\log_5 x)] = 0$.

So we must have:

$\log_5(\log_5 x) = 1$ or $\therefore \log_5 x = 5^1 = 5$

or $\therefore$ $x = 5^5 = 3125$