APPLICATIONS OF DE MOIVRE'S THEOREM

1 **(a)** Expand $(\cos\theta + i\sin\theta)^4$ by de Moivre's theorem and by the binomial theorem (Pascal's triangle) to show:

(i) $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$ **(ii)** $\sin 4\theta = 4\cos^3\theta\sin\theta - 4\sin^3\theta\cos\theta$

(b) Obtain an expression for $\tan 4\theta$ in terms of $\tan\theta$.

(c) By making suitable substitutions, solve the following. **(i)** $8x^4 - 8x^2 + 1 = 0$

(ii) $16x^4 - 16x^2 + 1 = 0$ **(iii)** $16x^4 - 16x^2 + 3 = 0$ **(iv)** $x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$

APPLICATIONS OF DE MOIVRE'S THEOREM

2 **(a)** Given that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ (see Example **19**, page 20), solve $8x^3 - 6x - 1 = 0$.

(b) Show that $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{8}$.

APPLICATIONS OF DE MOIVRE'S THEOREM

3 Let $z = \cos\theta + i\sin\theta$.

(a) Show that: **(i)** $z^n + z^{-n} = 2\cos n\theta$ **(ii)** $z^n - z^{-n} = 2i\sin n\theta$

(b) Show that $(z - z^{-1})^3 = (z^3 - z^{-3}) - 3(z - z^{-1})$

(c) Hence show that $\sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$.

APPLICATIONS OF DE MOIVRE'S THEOREM

4 **(a)** Let $z = \cos\theta + i\sin\theta$ and let $w = z + \frac{1}{z}$. Given $z^n + z^{-n} = 2\cos n\theta$, prove that $w^3 - 2w^2 - w + 2 = \left(z^3 + \frac{1}{z^3}\right) - 2\left(z^2 + \frac{1}{z^2}\right) + 2\left(z + \frac{1}{z}\right) - 2.$

(b) Hence solve $\cos 3\theta - 2\cos 2\theta + 2\cos\theta - 1 = 0$ for $-\pi \leq \theta \leq \pi$.

APPLICATIONS OF DE MOIVRE'S THEOREM

5 Express $\cos 3\theta$ and $\cos 2\theta$ in terms of $\cos\theta$. Show that the equation $\cos 3\theta = \cos 2\theta$ can be expressed as $4x^3 - 2x^2 - 3x + 1 = 0$, where $x = \cos\theta$. By solving this equation for x, find the exact value of $\cos\frac{2\pi}{5}$.

APPLICATIONS OF DE MOIVRE'S THEOREM

6 **(a)** Use de Moivre's theorem to express $\cos 4\theta$ in terms of $\cos\theta$.

(b) Use your result from part **(a)** to solve the equation $8x^4 - 8x^2 + 1 = 0$.

(c) Show that: $\cos\frac{\pi}{8} + \cos\frac{3\pi}{8} + \cos\frac{5\pi}{8} + \cos\frac{7\pi}{8} = 0$.

(d) Show that: $\cos\frac{\pi}{8}\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\cos\frac{7\pi}{8} = \frac{1}{8}$.

7 (a) Use de Moivre's theorem to express $\cos 5\theta$ and $\sin 5\theta$ as powers of $\cos\theta$ and $\sin\theta$.

(b) Hence express $\tan 5\theta$ as a rational function of t where $t = \tan\theta$.

(c) By considering the roots of $\tan 5\theta = 0$, deduce that $\tan\frac{\pi}{5}\tan\frac{2\pi}{5}\tan\frac{3\pi}{5}\tan\frac{4\pi}{5} = 5$.