Determine whether the scenario involves independent or dependent events.

1) There are eight nickels and eight dimes in your pocket. You randomly pick a coin out of your pocket and place it on a counter. Then you randomly pick another coin. Both coins are nickels.

DEPENDENT

2) A bag contains four red marbles and six blue marbles. You randomly pick a marble and then return it to the bag before picking another marble. Both the first and second marbles are red.

INDEPENDENT

3) You flip a coin twice. The first flip lands heads-up and the second flip lands tails-up.

INDEPENDENT

4) You roll a fair six-sided die three times. The die shows an even number every time.

INDEPENDENT

Find the probability.

5) A bag contains three red marbles and three blue marbles. You randomly pick a marble and then return it to the bag before picking another marble. The first marble is red and the second marble is blue.

$\frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$

6) A spinner has an equal chance of landing on each of its six numbered regions. You spin twice. The first spin lands in region one and the second spin lands in region three.

$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

7) There are seven boys and seven girls in a class. The teacher randomly selects one student to answer a question. Later, the teacher randomly selects a different student to answer another question. The first student is a boy and the second student is a girl.

$\frac{7}{14} \times \frac{7}{13} = \frac{49}{182} = \frac{7}{26}$

8) A basket contains three apples, four peaches, and three pears. You randomly select and eat three pieces of fruit. The first piece of fruit is an apple and the next two pieces are peaches.

$\frac{3}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{36}{720} = \frac{1}{20}$

Determine if events *A* and *B* are independent.

if they're independent, then $P(A \text{ and } B) = P(A) \times P(B)$

9) $P(A) = \frac{11}{20}$ $P(B) = \frac{1}{4}$ $P(A \text{ and } B) = \frac{11}{100}$

$\frac{11}{20} \times \frac{1}{4} = \frac{11}{80} \neq \frac{11}{100}$ so not independent (dependent)

10) $P(A) = \frac{1}{2}$ $P(B) = \frac{4}{5}$ $P(A \text{ and } B) = \frac{2}{5}$

$\frac{1}{2} \times \frac{4}{5} = \frac{4}{10} = \frac{2}{5}$ so independent

11) $P(A) = \frac{9}{20}$ $P(B) = \frac{3}{10}$ $P(A \text{ and } B) = \frac{27}{200}$

$\frac{9}{20} \times \frac{3}{10} = \frac{27}{200} = \frac{27}{200}$ so independent

12) $P(A) = \frac{7}{10}$ $P(\text{not } B) = \frac{7}{10}$ $P(A|B) = \frac{7}{20}$

so $P(B) = 3/10$

$\frac{7}{10} \times \frac{3}{10} = \frac{21}{100} \neq \frac{7}{20}$ so dependent

So $P(A \text{ and } B) = P(A) \times P(B)$

Events A and B are independent. Find the missing probability.

13) $P(B) = \frac{11}{20}$ $P(A \text{ and } B) = \frac{33}{80}$ $P(A) = ?$

$P(A) = \frac{P(A \text{ and } B)}{P(B)} = \frac{33/80}{11/20}$

$P(A) = \frac{33}{80} \times \frac{20}{11} = \frac{3}{4}$

14) $P(A) = \frac{13}{20}$ $P(B) = \frac{7}{20}$ $P(A \text{ and } B) = ?$

$P(A \text{ and } B) = \frac{13}{20} \times \frac{7}{20} = \frac{91}{400}$

15) $P(A) = \frac{7}{20}$ $P(B|A) = \frac{1}{4}$ $P(B) = ?$

$P(B) = P(B|A) = \frac{1}{4}$ as independents

~~$P(B) = \frac{1}{4} \times \frac{20}{1} = 5$~~

16) $P(A) = \frac{13}{20}$ $P(B) = \frac{7}{10}$ $P(B|A) = ?$

$P(B|A) = P(B)$ as A and B are independent events.

So $P(B|A) = \frac{7}{10}$

2 In a class of 25 boys and 15 girls, 8 boys and 7 girls wear glasses. One student from the class is selected. Indicate whether each statement below is correct or incorrect. 15/40

	glasses	no glasses	
Boys	8	17	25
Girls	7	8	15
	15	25	40

25/40

(a) $P(\text{boy}) = \frac{5}{8}$ yes.

(b) $P(\text{wears glasses}) = \frac{3}{8}$ yes.

(c) $P(\text{boy and wears glasses}) = \frac{1}{5}$

$\frac{8}{40} = \frac{1}{5}$ yes

(d) $P(\text{girl and does not wear glasses}) = \frac{7}{40}$

$\frac{8}{40}$ NO

8 In a large school, 25% of the students ride bicycles to school and 40% of the students have fair hair. One student is selected at random. What is the probability that the student:

(a) has fair hair and rides a bicycle to school

(b) does not have fair hair and does not ride a bicycle to school

(c) has fair hair but does not ride a bicycle to school

(d) rides a bicycle to school but does not have fair hair?

a) $0.4 \times 0.25 = 0.1$ 10%

b) $0.6 \times 0.75 = 0.45$ 45%

c) $0.4 \times 0.75 = 0.3$ 30%

d) $0.25 \times 0.6 = 0.15$ 15%

9 For a certain species of bird, there is a chance of 4 in 5 that a fledgling will survive the first month after birth. From a brood of 3 fledglings, what is the probability that:

(a) all will survive

$\left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) = \left(\frac{4}{5}\right)^3 = \frac{64}{125}$

(b) none will survive

$\left(\frac{1}{5}\right)^3 = \frac{1}{125}$

(c) at least one will survive?

$P(\text{at least one will survive}) = 1 - P(\text{none survives})$

$= 1 - \frac{1}{125}$

$= \frac{124}{125}$

12 An athlete competes in races over 100 m, 200 m and 400 m, and she estimates her chances of winning as $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Using these probabilities, calculate the probability that:

(a) she wins all three races
(b) she loses all three races
(c) she wins the 100 m race and loses the others
(d) she wins the 400 m race and loses the others
(e) she wins the 100 m race and the 200 m race but loses the 400 m race.

a) $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$ b) $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{6}{24} = \frac{1}{4}$

c) $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{6}{24} = \frac{1}{4}$ d) $\frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} = \frac{1}{12}$

e) $\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} = \frac{3}{24} = \frac{1}{8}$

18 The probability that a certain woman will be alive in 20 years is $\frac{2}{3}$ and the probability that a certain man will be alive is $\frac{3}{5}$. What is the probability that in 20 years' time:

(a) both will be alive
(b) only one will be alive
(c) at least one will be alive?

$\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$

$\frac{2}{3} \times \frac{2}{5} + \frac{1}{3} \times \frac{3}{5}$

$= \frac{7}{15}$

P(at least one alive) = 1 − P(none alive)

$= 1 - \frac{1}{3} \times \frac{2}{5} = 1 - \frac{2}{15}$

$= \frac{13}{15}$

20 On average, a student misses her bus to school once every eight weeks (where there are five days per school week). Find the probability that she:

40 days

(a) does not miss the bus on any one morning
(b) catches the bus on two successive mornings
(c) catches the bus each morning for a week (5 days)
(d) misses the bus on at least one morning in a week.

a) $\left(\frac{39}{40}\right)$ b) $\left(\frac{39}{40}\right)^2$

c) $\left(\frac{39}{40}\right)^5$

d) P(misses bus at least one morning in a week) = 1 − P(never misses that week)

$= 1 - \left(\frac{39}{40}\right)^5 \approx 0.12$ approx.

21 To open a locked safe requires a correct 3-digit combination. If a combination is chosen at random, calculate the probability of:

Numbers go from 0 to 9 (10 digits)

(a) succeeding at the first attempt
(b) failing at the first attempt.

$\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000}$

P(failing at first attempt) = 1 − P(succeeding at first attempt)

$= 1 - \frac{1}{1000}$

$= \frac{999}{1000}$

22 The first race at Randwick has 13 horses running and the second race has 16 horses. Assuming that all horses have an equal chance of winning, calculate the probability of predicting:

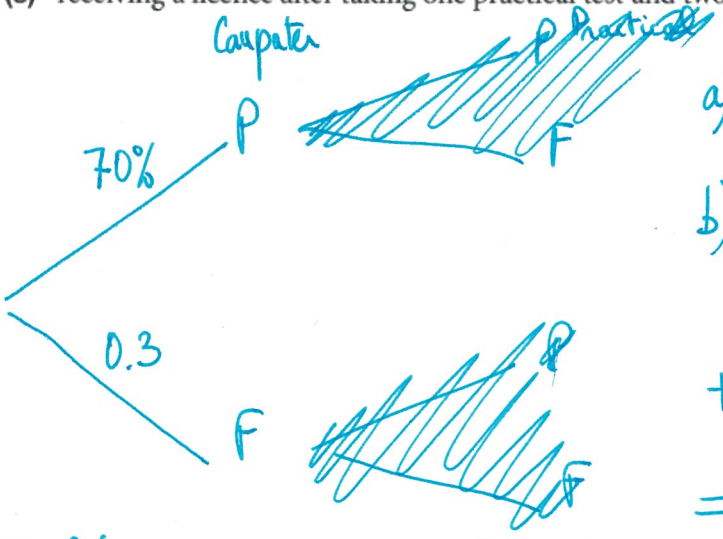
(a) a double (i.e. a winner in each race)
(b) a quinella (i.e. the two horses that come first and second, in either order) in the first race
(c) a quinella in the second race **(d)** a quinella in both races.

a) $\frac{1}{13} \times \frac{1}{16} = \frac{1}{208}$ b) $\left(\frac{1}{13} \times \frac{1}{12}\right) \times 2 = \frac{1}{78}$

c) $2\left(\frac{1}{16} \times \frac{1}{15}\right) = \frac{1}{120}$ d) $\frac{1}{78} \times \frac{1}{120} = \frac{1}{9360}$

23 To gain a driver licence in NSW, you must pass both a touch-screen computer-based test and a practical driving test. Statistics show that 70% of learners pass the computer-based test on the first attempt; of those who fail, 90% pass on the second attempt. Also, 60% of learners pass their first practical test and 80% pass their second practical test. The computer-based and practical tests are independent. Calculate the probability of:

(a) passing the computer-based test on the second attempt
(b) passing the computer-based test after no more than two attempts
(c) needing to take a third computer-based test
(d) passing the practical test on the second attempt
(e) receiving a licence after taking one practical test and two computer-based tests.



a) $0.3 \times 0.9 = 0.27$

b) P(computer test after exactly 1 or 2 attempts) = P(passing at 1st attempt) + P(passing at 2nd attempt)
$= 0.7 + 0.27 = 0.97$

c) P(needing 3rd test) $= 1 - 0.97 = 0.03$

d) P(passing practical test on 2nd attempt) $= 0.4 \times 0.8 = 0.32$

e) $P = 0.6 \times 0.27 = 0.162$