MOTION, FORCES AND PROJECTILES - CHAPTER REVIEW

1 A car is moving at a speed of $100\,\text{km}\,\text{h}^{-1}$ and a truck is moving at $90\,\text{km}\,\text{h}^{-1}$ in the opposite direction. The relative velocity of the car with respect to the truck is:

A $10\,\text{km}\,\text{h}^{-1}$ in the direction of the truck
B $10\,\text{km}\,\text{h}^{-1}$ in the direction of the car
C $190\,\text{km}\,\text{h}^{-1}$ in the direction of the truck
D $190\,\text{km}\,\text{h}^{-1}$ in the direction of the car

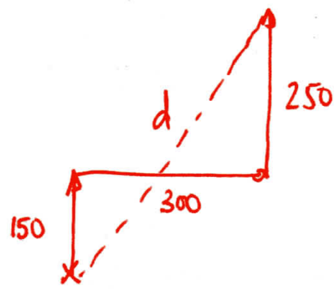
$\boxed{D}$

2 On his way home from training, Mitch walks along three streets. He walks 150 m north, 300 m east and 250 m north. The magnitude of Mitch's resultant displacement is:

A 200 m **B** 400 m $\boxed{C}$ 500 m **D** 700 m

$d^2 = 300^2 + (150 + 250)^2 = 250{,}000$

$\therefore d = 500\text{ m}$ Response $\boxed{C}$



3 Three forces are acting simultaneously in the same plane on an object. The resultant force is $\underset{\sim}{F} = 520\,\text{N}$ due east. Which of the following combinations of forces will give this resultant force?

A $F_1 = 350\,\text{N}$ west, $F_2 = 270\,\text{N}$ east, $F_3 = 440\,\text{N}$ west
B $F_1 = 350\,\text{N}$ east, $F_2 = 270\,\text{N}$ west, $F_3 = 440\,\text{N}$ east
C $F_1 = 350\,\text{N}$ east, $F_2 = 270\,\text{N}$ west, $F_3 = 440\,\text{N}$ west
D $F_1 = 350\,\text{N}$ west, $F_2 = 270\,\text{N}$ east, $F_3 = 440\,\text{N}$ east

East is the positive direction
West (negative)

520 N

$\boxed{A}$ is $-350 + 270 - 440 = -20\text{ N}$ (West) so NO

$\boxed{B}$ is $350 - 270 + 440 = 520\text{ N}$ (East) YES.

$\boxed{C}$ is $350 - 270 - 440 = -360\text{ N}$ (West) NO

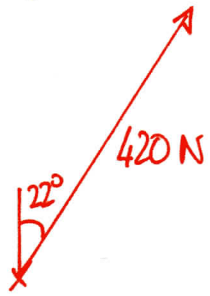
$\boxed{D}$ is $-350 + 270 + 440 = 360\text{ N}$ (East) NO)

so it's only $\boxed{B}$

4 A soccer ball is struck with a force of 420 N in the direction N22°E. The northerly and easterly components of this force are $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$ respectively. The magnitudes of these forces are:

A $\underset{\sim}{F}_1 = 87\text{ N}, \underset{\sim}{F}_2 = 411\text{ N}$ **B** $\underset{\sim}{F}_1 = 411\text{ N}, \underset{\sim}{F}_2 = 87\text{ N}$

C $\underset{\sim}{F}_1 = 157\text{ N}, \underset{\sim}{F}_2 = 389\text{ N}$ **D** $\underset{\sim}{F}_1 = 389\text{ N}, \underset{\sim}{F}_2 = 157\text{ N}$



Horizontal component is $420 \sin 22 = 157$ N

Vertical component is $420 \cos 22 = 389$ N

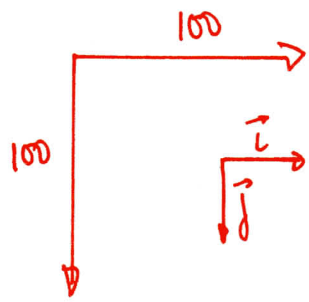
So $\boxed{D}$

5 Forces are acting on an object. Calculate the resultant force acting on the object for each of the following:

(a) $\underset{\sim}{F}_1 = 250\text{ N north}, \underset{\sim}{F}_2 = 450\text{ N south}, \underset{\sim}{F}_3 = 125\text{ N north}$ **(b)** $\underset{\sim}{F}_1 = 100\text{ N east}, \underset{\sim}{F}_2 = 100\text{ N south}$

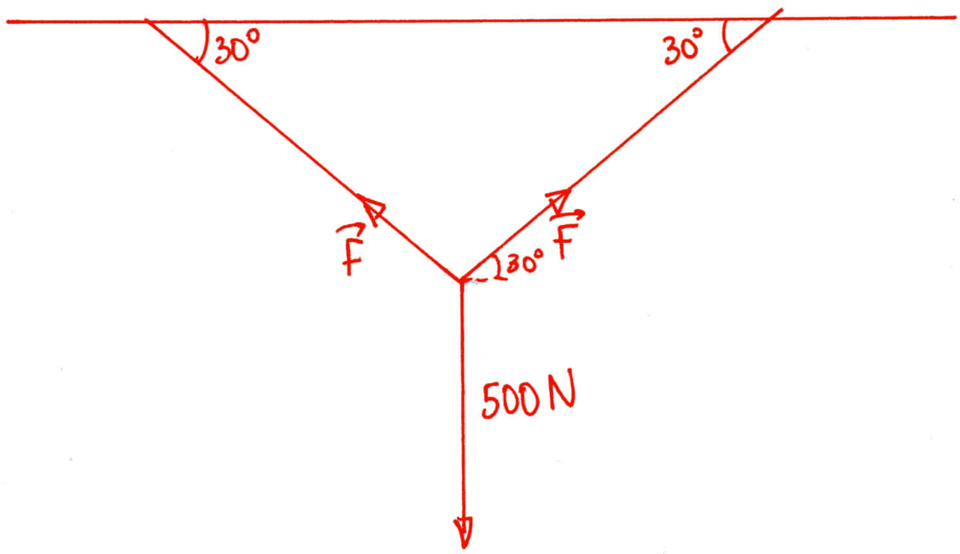
a) $\sum_i F_i = 250 - 450 + 125 = -75$ so 75 N South

b) $\sum_i F_i = 100\vec{i} + 100\vec{j}$



or $\sqrt{2} \times 100$ Newton in the direction SE.

6 A particle of mass 50 kg is suspended by two strings attached to two points in the same horizontal plane. If the two strings each make angles of 30° respectively to the horizontal, find the magnitude of the tension in each string, in newtons, given that $\sin 30° = \frac{1}{2}$.



Weight $= 50 \times 10 = 500$ N. assuming $g = 10\ ms^{-2}$

The vertical equilibrium of the forces must be zero, i.e.

$500 = |\vec{F}| \sin 30 + |\vec{F}| \sin 30$

OR $500 = 2|\vec{F}| \sin 30$

$500 = |\vec{F}|$ hence $|\vec{F}| = 500$ N.

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7 Jaide paddles her canoe in a direction perpendicular to the riverbanks. The river is 120 m wide. The canoe's speed in this direction is 1.5 m s^{-1}. The water in the river is flowing at 0.8 m s^{-1} and you can assume that the riverbanks are parallel straight lines.

(a) Calculate the velocity of the canoe relative to the bank. Give the angle correct to two decimal places.

(b) At what distance, in the direction downstream from the starting point, will Jaide get to the other riverbank?

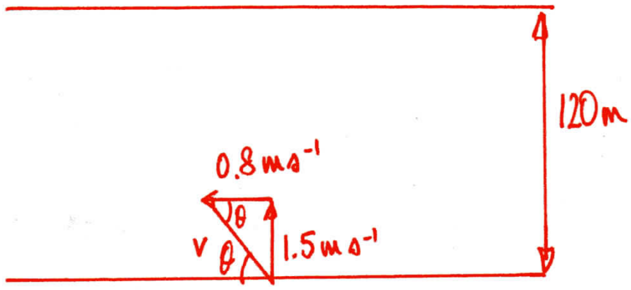
(c) Calculate Jaide's final displacement from the starting point.

(d) Jake is going to cross the river in another canoe. He can paddle at the speed of 1.8 m s^{-1} relative to the water's flow. At what direction should Jake row to get to the same end-point as Jaide, and how much time will this take?

a) $V^2 = 0.8^2 + 1.5^2$

$\therefore V = 1.7 \text{ ms}^{-1}$

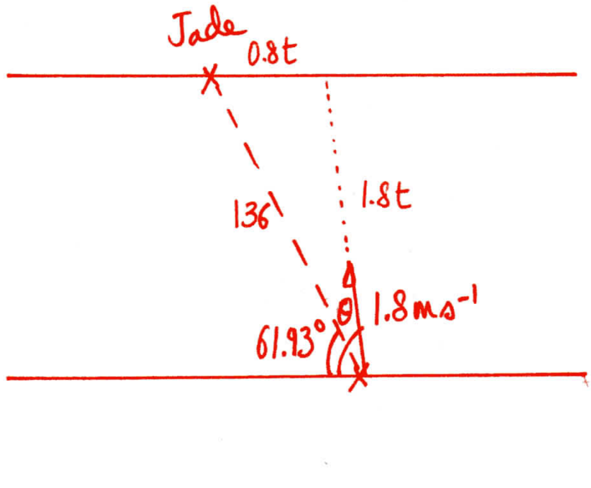
With $\tan\theta = \frac{1.5}{0.8}$ so $\theta = 61.93°$



b) it takes her $\frac{120}{1.5} = 80$ s to get to the other side.
She would have travelled down, during that time, $80 \times 0.8 = 64$ m downstream.

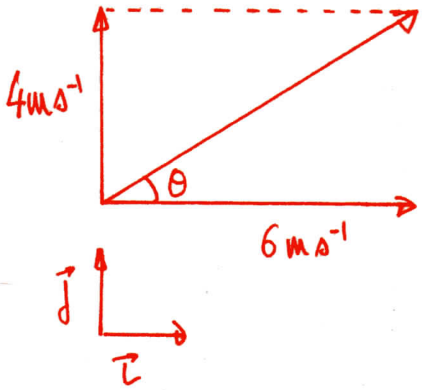
c) Final displacement $= \sqrt{64^2 + 120^2} = 136$ m

d) ?



8 A particle is projected with a velocity whose horizontal and vertical components are $6\,\mathrm{m\,s^{-1}}$ and $4\,\mathrm{m\,s^{-1}}$ respectively. Use $g = 10\,\mathrm{m\,s^{-2}}$.

(a) Draw a diagram to show this information.
(b) What is the angle of projection?
(c) Find expressions for $\underset{\sim}{v}(t)$ and $\underset{\sim}{r}(t)$.
(d) Find the greatest height reached.
(e) Find the horizontal range.



b) $\tan\theta = \dfrac{4}{6} = \dfrac{2}{3} \quad \therefore\ \theta = 33.69°$

c) $\vec{v}(0) = 6\vec{i} + 4\vec{j}$

$\vec{a} = -g\vec{j}$

$\therefore\ \vec{v}(t) = -gt\vec{j} + \vec{C} \qquad$ at $t=0$ $\vec{v}(0) = 6\vec{i} + 4\vec{j}$.

Hence $\vec{v}(t) = -gt\vec{j} + 6\vec{i} + 4\vec{j} = [-gt+4]\vec{j} + 6\vec{i}$

$\therefore\ \vec{r}(t) = \left[-\frac{1}{2}gt^2 + 4t\right]\vec{j} + 6t\,\vec{i} + \vec{K}$

At $t=0$, $\vec{r}(0) = \vec{0}$ hence $\vec{K} = \vec{0}$

$\vec{r}(t) = 6t\,\vec{i} + \left[-\frac{1}{2}gt^2 + 4t\right]\vec{j} = 6t\,\vec{i} + \left[-5t^2 + 4t\right]\vec{j}$

d) the greatest height is reached when the vertical speed is 0,

i.e. $-10t + 4 = 0 \Rightarrow t = \dfrac{4}{10} = \dfrac{2}{5} = 0.4\,s$.

At $t = 0.4\,s$, the vertical position is $\left[-5\times 0.4^2 + 4\times 0.4\right] = 0.8\,m$

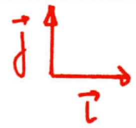
e) The horizontal range is reached when $y = 0$, i.e.

$-5t^2 + 4t = 0$, which occurs at either $t=0$ or $t = 4/5 = 0.8\,s$

At $t = 0.8\,s$, $x(0.8) = 6 \times 0.8 = 4.8\ m$

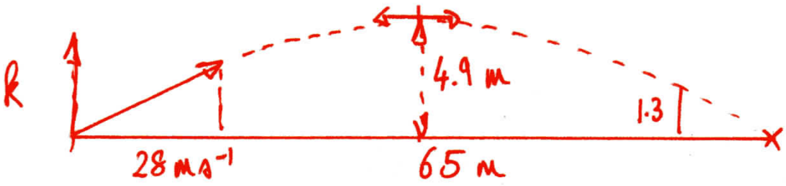
9 A cricketer hits a cricket ball off the ground towards a fielder who is 65 m away. The ball reaches a maximum height of 4.9 m and the horizontal component of the velocity is 28 m s^{-1}. Use $g = 9.8$ m s^{-2}.

- **(a)** Find expressions for $\underset{\sim}{v}(t)$ and $\underset{\sim}{r}(t)$.
- **(b)** How much time does it take for the ball to reach its greatest height?
- **(c)** How far has the ball travelled horizontally when it has descended to a height of 1.3 m?
- **(d)** Find the constant speed with which the fielder must run forward, starting at the instant the ball is hit, in order to catch the ball at a height of 1.3 m above the ground.



a) $\vec{a} = -g\vec{j}$

$\vec{v}(t) = -gt\vec{j} + \vec{C}$



At $t=0$ $\vec{v}(0) = 28\vec{i} + k\vec{j}$

$\therefore \vec{v}(t) = -gt\vec{j} + 28\vec{i} + k\vec{j} = 28\vec{i} + [k - gt]\vec{j} = 28\vec{i} + [k - 9.8t]\vec{j}$

Hence $\vec{r}(t) = 28t\vec{i} + [kt - \frac{1}{2}gt^2]\vec{j} + \vec{K}$

At $t=0$, $\vec{r}(0) = \vec{0}$, so $\vec{K} = \vec{0}$ $\therefore \vec{r}(t) = 28t\vec{i} + [kt - \frac{1}{2}gt^2]\vec{j}$

When $t = \frac{k}{9.8}$, then $y = 4.9$ therefore: $k \times \frac{k}{9.8} - \frac{1}{2} \times 9.8 \times \left(\frac{k}{9.8}\right)^2 = 4.9$

Hence $k^2\left[\frac{1}{9.8} - \frac{1}{2 \times 9.8}\right] = 4.9 \Rightarrow k^2 = 96.04 \Rightarrow k = 9.8$.

So $\vec{v}(t) = 28\vec{i} + 9.8[1 - t]\vec{j}$ and $\vec{r}(t) = 28t\vec{i} + t[9.8 - 4.9t]\vec{j}$

b) greatest height at $t = \frac{k}{9.8} = \frac{9.8}{9.8} = 1$ s.

c) $y = 1.3$ when $t[9.8 - 4.9t] = 1.3 \Leftrightarrow -4.9t^2 + 9.8t - 1.3 = 0$

$\Delta = 70.56$ $t = \frac{-9.8 \pm \sqrt{70.56}}{2 \times (-4.9)} = 1.857$ s.

Then, at that time, $x(1.857) = 28 \times 1.857 = 52$ m

d) So the fielder must run $(65 - 52) = 13$ m in 1.857 s,

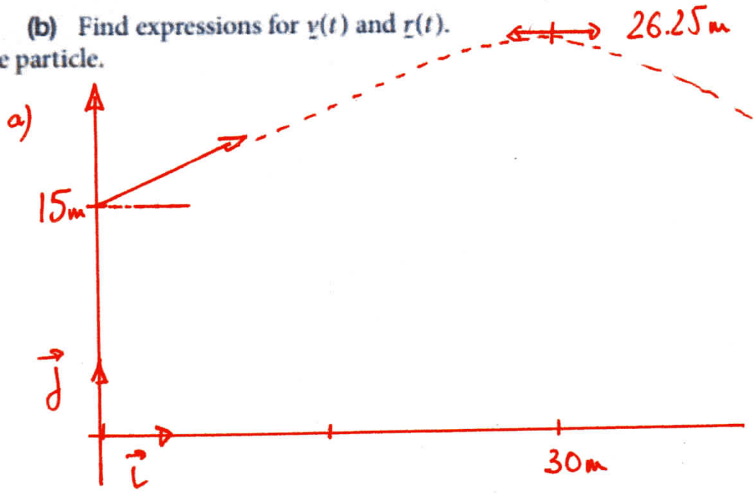
i.e. a speed of 7 m s^{-1}

10 A particle is projected from a point 15 m above horizontal ground. At its highest point it just clears the top of a wall 26.25 m high and 30 m away. Use $g = 10\,\text{m s}^{-2}$.

(a) Draw a diagram to show this information. **(b)** Find expressions for $\underset{\sim}{v}(t)$ and $\underset{\sim}{r}(t)$.

(c) Find the speed and the angle of projection of the particle.

a)



b) $\vec{a} = -g\vec{j} = -10\vec{j}$

$\vec{v}(t) = -10t\vec{j} + \vec{C}$

At $t=0$, $\vec{v}(0) = V_{xo}\vec{i} + V_{yo}\vec{j}$

So $\vec{v}(t) = V_{xo}\vec{i} + [V_{yo} - 10t]\vec{j}$

$\vec{r}(t) = V_{xo}t\vec{i} + [V_{yo}t - 5t^2]\vec{j} + \vec{K}$

At $t=0$ $\vec{r}(0) = 15\vec{j}$ so $\vec{K} = 15\vec{j}$

$\vec{r}(t) = V_{xo}t\vec{i} + [V_{yo}t - 5t^2 + 15]\vec{j}$

So $\begin{cases} x(t) = V_{xo}t \\ y(t) = V_{yo}t - 5t^2 + 15 \end{cases} \Leftrightarrow \begin{cases} t = \dfrac{x}{V_{xo}} \\ y = V_{yo}\dfrac{x}{V_{xo}} - \dfrac{5x^2}{V_{xo}^2} + 15 \end{cases}$

So when $t = V_{yo}/10$, $y = 26.25 = V_{yo} \times \dfrac{V_{yo}}{10} - 5\left(\dfrac{V_{yo}}{10}\right)^2 + 15$.

$\therefore V_{yo}^2\left[\dfrac{1}{10} - \dfrac{5}{100}\right] = 26.25 - 15 = 11.25$

$\therefore V_{yo}^2 = \dfrac{11.25}{0.05}$ $\therefore V_{yo} = 15\,\text{ms}^{-1}$

Therefore, as when $x = 30$, $y = 26.25 = \dfrac{15 \times 30}{V_{xo}} - 5 \times \dfrac{225}{100} + 15$

$\therefore \dfrac{450}{V_{xo}} = 22.5$ so $V_{xo} = 20\,\text{ms}^{-1}$

Therefore $\vec{r}(t) = 20t\vec{i} + [15t - 5t^2 + 15]\vec{j}$

c) Speed $= \sqrt{15^2 + 20^2} = 25\,\text{ms}^{-1}$

$\tan\theta = \dfrac{V_{yo}}{V_{xo}} = \dfrac{15}{20} = \dfrac{3}{4}$ so $\theta \simeq 36.87°$

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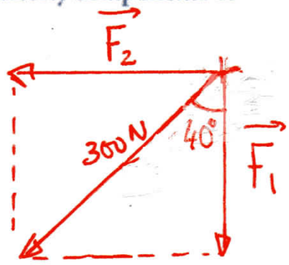
11 A tennis ball is struck with a force of 300 N in the direction S40°W. The southerly and westerly components of this force are $\underset{\sim}{F_1}$ and $\underset{\sim}{F_2}$ respectively. The magnitudes of these forces are:



A $F_1 = 230\,\text{N}, F_2 = 230\,\text{N}$

B $F_1 = 193\,\text{N}, F_2 = 230\,\text{N}$

C $F_1 = 230\,\text{N}, F_2 = 193\,\text{N}$

D $F_1 = 150\,\text{N}, F_2 = 260\,\text{N}$

$|\vec{F_1}| = 300 \cos 40 = 230\text{N}$

$|\vec{F_2}| = 300 \sin 40 = 193\text{ N}$

So Response $\boxed{C}$