PROBLEM SOLVING WITH DERIVATIVES

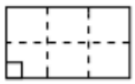
1 A rectangular block of land is enclosed by 160 m of fencing. If the breadth of the block is x m:

(a) express the length of the block in terms of x

(b) find the function $A(x)$ for the area of the block

(c) find the maximum area of the block that can be fenced using this fencing.

4 A rectangular paddock is to be fenced and also divided into six smaller rectangular paddocks, with one dividing fence parallel to the length and two dividing fences parallel to the breadth (as shown in the diagram by dashed lines). The total length of fencing to be used is 120 m. If the width of the paddock is x m and the breadth is y m, find:



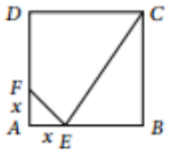
(a) an expression giving y as a function of x

(b) the function $A(x)$ for the area of the original paddock

(c) the maximum possible area of the paddock.

(d) To allow access to the paddocks, six gates are to be added to the fences. Each gate is 3 m wide. What is the new maximum area of the large paddock?

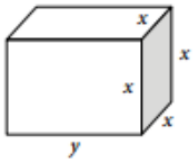
5 $ABCD$ is a square of unit length. Points E and F are on the sides AB and AD respectively so that $AE = AF = x$.



(a) Express the area of the quadrilateral $CDFE$ as a function of x.

(b) Find the greatest area that the quadrilateral can have.

7 A block of wood in the shape of a cuboid has square ends of edge length x cm. The length of the block is y cm. The sum of length of the block and the perimeter of one end is 12 cm.

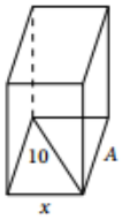


(a) Express y in terms of x.

(b) Find the volume V as a function of x.

(c) What is the largest possible volume of the block?

12 The diagonal of the base of a box in the shape of a cuboid has a length of 10 cm. One edge of the base has a length of x cm, as shown in the diagram.

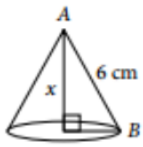


(a) Express, in terms of x, the length of the other edge of the base.

(b) The height of the box is equal to the length of this other edge. Find the volume of the box in terms of x.

(c) Calculate the maximum volume of the box.

13 The slant edge AB of a right circular cone is 6 cm. The vertical height of the cone is x cm, as shown in the diagram.



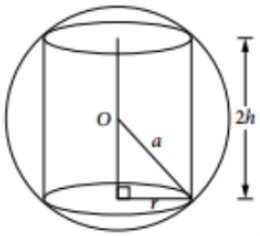
(a) Express the radius of the base in terms of x.

(b) Express the volume of the cone in terms of x.

(c) Find the vertical height of the cone when the volume is a maximum.

16 A piece of wire of length 50 cm is cut into two sections. One section is used to construct a rectangle whose dimensions are in the ratio 3 : 1; the other section is used to construct a square. Find the dimensions of the rectangle and the square so that the total enclosed area is a minimum.

20 A cylinder is inscribed in a sphere of radius a, centred at O. The height of the cylinder is $2h$ and the radius of the base is r, as shown in the diagram.



(a) Show that the volume V of the cylinder is given by $V = 2\pi r^2\sqrt{a^2 - r^2}$.

(b) Find the value of r for which the volume of the cylinder is a maximum. Explain why your value of r gives the maximum volume.