THE DEFINITE INTEGRAL AND THE PRIMITIVE FUNCTION

1 Evaluate $\int_0^3 x^2\,dx$, $\int_3^0 x^2\,dx$ and show that $\int_0^3 x^2\,dx = -\int_3^0 x^2\,dx$.

$$\int_0^3 x^2\,dx = \left[\frac{x^3}{3}\right]_0^3 = \frac{3^3}{3} - \frac{0^3}{3} = 9$$

$$\int_3^0 x^2\,dx = \left[\frac{x^3}{3}\right]_3^0 = \frac{0^3}{3} - \frac{3^3}{3} = -9$$

$$\therefore \int_0^3 x^2\,dx = -\int_3^0 x^2\,dx$$

2 Evaluate $\int_1^2 x^2\,dx$, $\int_2^3 x^2\,dx$, $\int_1^3 x^2\,dx$ and show that $\int_1^3 x^2\,dx = \int_1^2 x^2\,dx + \int_2^3 x^2\,dx$.

$$\int_1^2 x^2\,dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{7}{3}$$

$$\int_2^3 x^2\,dx = \left[\frac{x^3}{3}\right]_2^3 = \frac{3^3}{3} - \frac{2^3}{3} = \frac{19}{3}$$

$$\int_1^3 x^2\,dx = \left[\frac{x^3}{3}\right]_1^3 = \frac{3^3}{3} - \frac{1^3}{3} = \frac{26}{3}$$

As $\frac{26}{3} = \frac{7}{3} + \frac{19}{3}$

$$\therefore \int_1^3 x^2\,dx = \int_1^2 x^2\,dx + \int_2^3 x^2\,dx$$

5 Evaluate: **(a)** $\int_0^1 (2x-5)\,dx$ **(b)** $\int_{-1}^4 (3x+1)\,dx$ **(c)** $\int_{-2}^2 (x^2-4)\,dx$

a) $\int_0^1 (2x-5)\,dx = \left[x^2-5x\right]_0^1 = (1^2-5)-(0^2-5\times 0) = -4$

b) $\int_{-1}^4 (3x+1)\,dx = \left[\frac{3x^2}{2}+x\right]_{-1}^4 = \left(\frac{3\times 16}{2}+4\right)-\left(\frac{3\times(-1)^2}{2}+(-1)\right)$

$= 28 - 0.5 = 27.5$

c) $\int_{-2}^2 (x^2-4)\,dx = \left[\frac{x^3}{3}-4x\right]_{-2}^2 = \left(\frac{8}{3}-8\right)-\left(\frac{(-2)^3}{3}-4\times(-2)\right)$

$= -\frac{16}{3} - \left[\frac{16}{3}\right]$

$= -\frac{32}{3} = -10\frac{2}{3}$

THE DEFINITE INTEGRAL AND THE PRIMITIVE FUNCTION

Evaluate (g) $\int_{-8}^{8} x^{\frac{2}{3}}\,dx$ (h) $\int_{0}^{1}\left(x^{\frac{1}{5}} - x^{\frac{1}{3}}\right)dx$ (i) $\int_{0}^{1}\left(\sqrt{x} - \sqrt[3]{x}\right)dx$

g) $\int_{-8}^{8} x^{2/3}\,dx = \left[\frac{x^{\frac{2}{3}+1}}{\frac{5}{3}}\right]_{-8}^{8} = \left[\frac{3}{5}x^{5/3}\right]_{-8}^{8} = \frac{3}{5}\left[8^{5/3} - (-8)^{5/3}\right]$

$= \frac{3}{5}\left[(2^3)^{5/3} - ((-2)^3)^{5/3}\right] = \frac{3}{5}\left[2^5 - (-2)^5\right]$

$= \frac{3}{5}\left[2^5 + 2^5\right] = \frac{6 \times 2^5}{5} = \frac{192}{5}$

h) $\int_{0}^{1} x^{1/5} - x^{1/3}\,dx = \left[\frac{x^{\frac{1}{5}+1}}{\frac{6}{5}} - \frac{x^{\frac{1}{3}+1}}{\frac{4}{3}}\right]_{0}^{1} = \left[\frac{5}{6}x^{6/5} - \frac{3}{4}x^{4/3}\right]_{0}^{1}$

$= \frac{5}{6} - \frac{3}{4} = \frac{1}{12}$

i) $\int_{0}^{1}\left(\sqrt{x} - \sqrt[3]{x}\right)dx = \int_{0}^{1}\left(x^{1/2} - x^{1/3}\right)dx = \left[\frac{x^{\frac{1}{2}+1}}{\frac{3}{2}} - \frac{x^{\frac{1}{3}+1}}{\frac{4}{3}}\right]_{0}^{1}$

$= \left[\frac{2}{3}x^{3/2} - \frac{3}{4}x^{4/3}\right]_{0}^{1}$

$= \frac{2}{3} - \frac{3}{4} = -\frac{1}{12}$

13 If $\int_0^a (4-2x)dx = 4$, find the value of a.

14 If $\int_{-1}^a x\,dx = 0$, find the value of a.

13) $\int_0^a (4-2x)\,dx = \left[4x - x^2\right]_0^a = 4a - a^2$

But $4a - a^2 = 4 \iff a^2 - 4a + 4 = 0$

$\iff (a-2)^2 = 0$ so $a = 2$

14) $\int_{-1}^a x\,dx = \left[\frac{x^2}{2}\right]_{-1}^a = \frac{a^2}{2} - \frac{1}{2} = \frac{a^2-1}{2}$

But $\frac{a^2-1}{2} = 0$ so $a = 1$ or $a = -1$