# CIRCULAR AND SIMULTANEOUS INEQUALITIES

A circle divides the number plane into two regions, a finite region called its **interior** and an infinite region called its **exterior**, as well as the set of points that make up the circle.

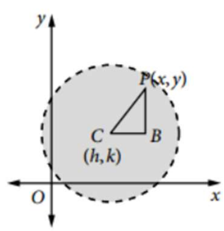
A parabola, cubic, quartic or hyperbola curve divides the number plane into two infinite regions, as well as the set of points that make up the curve.

## Points inside and outside a circle

A circle divides the number plane into three sets of points: the sets of points **on** the circle, **inside** the circle and **outside** the circle. The set of points on a circle of centre $C(h, k)$ and radius $r$ is given by the equation $(x-h)^2+(y-k)^2=r^2$.

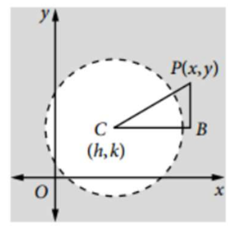
A point $P(x, y)$ lies on this circle if $CP = r$.

If $CP < r$, the point $P$ is inside the circle.




The graph of $(x-h)^2+(y-k)^2<r^2$ gives the interior of the circle.

If $CP > r$, the point $P$ is outside the circle.




The graph of $(x-h)^2+(y-k)^2>r^2$ gives the exterior of the circle.

## Regions involving simultaneous inequalities

### Example 8

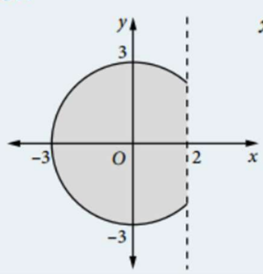
For the circle with centre $(0, 0)$ and radius 3 units, sketch the region of the Cartesian plane that includes all points on or inside the circle that are also:

**(a)** to the left of the line $x = 2$ **(b)** on or above the line $x + y = 3$.

In each case give the inequalities that define the region.

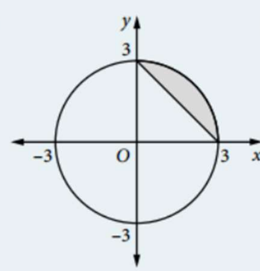
### Solution

**(a)**




$x^2+y^2 \le 9, x < 2$

**(b)**



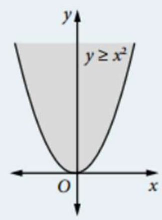

$x^2+y^2 \le 9, x + y \ge 3$

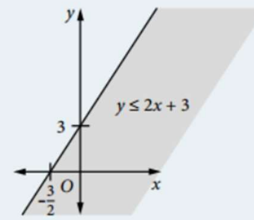
## Example 9

Sketch the region defined by $y \geq x^2$ and $y \leq 2x + 3$. Describe this region in words.

### Solution

To find the points of intersection, solve simultaneously the equations $y = x^2$ and $y = 2x + 3$.




This gives: $x^2 = 2x + 3$

$x^2 - 2x - 3 = 0$

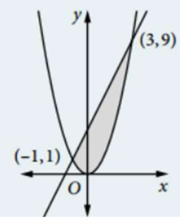
Factorise: $(x+1)(x-3) = 0$

Solve: $x = -1, 3$

Substitute into $y = 2x + 3$: $x = -1, y = 1$; $x = 3, y = 9$

Hence the points of intersection are $(-1, 1)$ and $(3, 9)$.

The shaded region is the points on and above the parabola $y = x^2$ that are also on or below the line $y = 2x + 3$.




## Example 10

Describe the region of the $x$–$y$ plane whose points satisfy the inequalities $y < 2 + x - x^2$ and $y + 2x \leq 2$.

### Solution

The graph of $y = 2 + x - x^2$ can be obtained by completing a table of values and then plotting points. It can also be obtained by completing the square for $x$ and then graphing according the shape and properties of $y = x^2$:

$$2 + x - x^2 = 2 - \left(x^2 - x\right)$$
$$= 2 + \tfrac{1}{4} - \left(x^2 - x + \tfrac{1}{4}\right)$$
$$= 2\tfrac{1}{4} - \left(x - \tfrac{1}{2}\right)^2$$

Hence you can graph $y = 2\frac{1}{4} - \left(x - \frac{1}{2}\right)^2$, which is the graph of $y = x^2$ turned upside down, moved 0.5 units to the right and moved 2.25 units up:

$y < 2 + x - x^2$ is the region below this curve.

$y + 2x \leq 2$ is the region on or below the line $y + 2x = 2$.

As shown in the diagram, the required region is the region on or below the line $y + 2x = 2$ that is contained between the 'arms' of $y = 2 + x - x^2$.