Question 1: Prove by mathematical induction that for all positive integer values of n:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Step 1: When $n = 1$, the left-hand side (LHS) is $LHS = 1$, whereas the right-hand side (RHS) is $RHS = \frac{1(1+1)}{2} = \frac{2}{2} = 1$.

Therefore the statement is true for $n = 1$

Step 2: Let assume that the statement is true for $n = k$, i.e.

$$1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}$$

In that case, for $k + 1$, the LHS of the equality is:

$LHS = 1 + 2 + 3 + \cdots + k + (k+1)$

$LHS = \dfrac{k(k+1)}{2} + (k+1)$ using the assumption above

$LHS = \dfrac{k(k+1)}{2} + \dfrac{2(k+1)}{2}$

$LHS = \dfrac{k(k+1) + 2(k+1)}{2}$

$LHS = \dfrac{(k+1)(k+2)}{2}$ which is the same statement as the assumption above, for $(k+1)$

Therefore, if the statement is true for k, then it is true for $(k+1)$

Step 3:

The statement is true for $n = 1$

The statement is true for $(k+1)$ if it is true for k.

Therefore, by induction, it is true for all $n \geq 1$

Question 2: Prove by mathematical induction that for all positive integer values of n:

$$1 + 2 + 2^2 + \cdots + 2^{n-1} = 2^n - 1$$

Step 1: When $n = 1$, the left-hand side (LHS) is $LHS = 1$, whereas the right-hand side (RHS) is $RHS = $ _____.

Therefore the _____ is _____ for $n = 1$

Step 2: Let _____ that the _____ is _____ for $n = k$, i.e.

$$1 + 2 + 2^2 + \cdots + 2^{k-1} = 2^k - 1$$

In that _____, for $k + 1$, the LHS of the _____ is:

$LHS = 1 + 2 + 2^2 + \cdots + 2^{k-1} + 2^k$

$LHS = (2^k - 1) + 2^k$ _____ the _____ above

$LHS = 2 \times 2^k - 1$

$LHS = 2^{k+1} - 1$

$LHS = 2^{k+1} - 1$ _____ is the same _____ as the
_____ above, for $(k + 1)$

_____, if the _____ is true for k, then it is true for _____

Step ____:

The _____ is true for $n = 1$

The statement is _____ for $(k + 1)$ if it is _____ for k.

Therefore, by _____, it is _____ for _____ $n \geq 1$

Question 3: Prove by mathematical induction that for all positive integer values of n:

$$1 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Step 1: When $n = 1$, the _____(LHS) is $LHS = 1$, whereas the
_____ (RHS) is $RHS = \frac{1(1+1)(2\times1+1)}{6} = \frac{2\times3}{6} = 1$.

_____ the _____ is _____ for $n = 1$

Step 2: Let _____ that the _____ is _____ for $n = k$, i.e.

$$1 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

In that _____, for $k + 1$, the _____ of the _____ is:

$LHS = 1 + 2^2 + 3^2 + \cdots + k^2 + (k+1)^2$

$LHS = \dfrac{k(k+1)(2k+1)}{6} + (k+1)^2$ _____ the assumption _____

$LHS = \dfrac{k(k+1)(2k+1)}{6} + \dfrac{6(k+1)^2}{6}$

$LHS = \dfrac{k(k+1)(2k+1) + 6(k+1)^2}{6}$

$LHS = \dfrac{(k+1)[k(2k+1) + 6(k+1)]}{6}$

$LHS = \dfrac{(k+1)(2k^2 + 7k + 6)}{6}$

$LHS = \dfrac{(k+1)(k+2)(2k+3)}{6}$

$LHS = \dfrac{(k+1)(k+2)[2(k+1)+3]}{6}$ _____ is the _____ statement as the
                                                  _____ above, for $(k+1)$

_____, if the _____ is true for____, then it is _____ for _____

Step 3:

The _____ is true for _____

The _____ is _____ for $(k+1)$ if ____ is _____ for ____

_____, by _____, it is _____ for _____

Question 4: Prove by mathematical induction that for all positive integer values of n:

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

Step 1: _____ $n = 1$, the _____ (LHS) is $LHS = \frac{1}{1 \times 3}$, _____ the

_____ (RHS) is $RHS = \frac{1}{2 \times 1 + 1} = \frac{1}{3}$.

_____ the _____ is true for $n = 1$

Step 2: _____ _____ that the _____ is _____ for $n = k$, i.e.

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \cdots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$$

In that _____, for $k + 1$, the _____ of the _____ is:

$$LHS = \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \cdots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{[2(k+1)-1][2(k+1)+1]}$$

$$LHS = \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \cdots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2k+1)(2k+3)}$$

$$LHS = \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$ _____ the _____ above

$$LHS = \frac{k(2k+3)}{(2k+1)(2k+3)} + \frac{1}{(2k+1)(2k+3)}$$

$$LHS = \frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$LHS = \frac{2k^2 + 3k + 1}{(2k+1)(2k+3)}$$

$$LHS = \frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$$

$$LHS = \frac{k+1}{2k+3}$$

$$LHS = \frac{k+1}{2(k+1)+1}$$ _____ is the same _____as the _____ _____, for _____

_____, ____ the _____ is _____ for ____, _____ it is ____ for _____

Step 3:

The _____ is _____ for _____

The _____ is _____ for _____if it is _____ for _____.

_____, by _____, it is _____ for all _____