LOGARITHMS – DEFINITION & LAWS OF LOGARITHMS

A logarithm (abbreviation "log") of a number to a given base is the power to which the base is raised to give the number.

Example 1: $2^4 = 16$ therefore $\log_2 16 = 4$

Example 2: $10^3 = 1000$ therefore $\log_{10} 1000 = 3$

Example 3: $5^3 = 125$ therefore $\log_5 125 = 3$

The base **a** is written as a subscript to the operator 'log', i.e. $\log_a$

So, if $a^n = x$ then $\log_a x = n$ We say '*log to the base* ***a*** *of x equals n*'.

First, we note that: $\boldsymbol{\log_a a = 1}$ as $a^1 = a$

And also: $\boldsymbol{\log_a 1 = 0}$ as $a^0 = 1$

The **laws of logarithms** are deduced from the index laws:

Let $a^n = x$ (and so $\log_a x = n$) and $a^m = y$ (and so $\log_a y = m$)

Index laws	Logarithms laws
$\boldsymbol{xy = a^n \times a^m = a^{n+m}}$	$\log_a(xy) = n + m = \log_a x + \log_a y$ therefore $\boldsymbol{\log_a(xy) = \log_a x + \log_a y}$ Example: $\log_2(35) = \log_2 5 + \log_2 7$
$\boldsymbol{\frac{x}{y} = a^n \div a^m = a^{n-m}}$	$\log_a\left(\frac{x}{y}\right) = n - m = \log_a x - \log_a y$ therefore $\boldsymbol{\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y}$ Example: $\log_2(7) = \log_2 42 - \log_2 6$
$\boldsymbol{\frac{1}{x} = \frac{1}{a^n} = a^{-n}}$	$\log_a\left(\frac{1}{x}\right) = -n = -\log_a x$ therefore $\boldsymbol{\log_a\left(\frac{1}{x}\right) = -\log_a x}$ Example: $\log_2\left(\frac{1}{32}\right) = -\log_2 32$
$\boldsymbol{x^p = (a^n)^p = a^{np}}$	$\log_a(x^p) = np = pn = p\log_a x$ therefore $\boldsymbol{\log_a(x^p) = p \times \log_a x}$ Example: $\log_2(81) = \log_2(3^4) = 4\log_2 3$

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The calculator can only calculate:

- log_{10} (which is abbreviated "*log*" not showing that it is in fact "*log of base 10*"); and
- log_e (abbreviated "*ln*" (i.e. "*logarithm naturalis*") not showing it's in fact "*log of base e*")

So we need to find a way to calculate logarithms for other bases.

For this purpose, we use the **change of base rule**.

Change of base rule:

Let $\log_a x = n$ which is equivalent to $a^n = x$

Taking the logarithm to base b on both sides, we obtain: $\log_b a^n = \log_b x$

or $n \log_b a = \log_b x$ (as $\log_b(a^n) = n \times \log_b a$, from the previous page)

so $n = \dfrac{\log_b x}{\log_b a}$

therefore $\boldsymbol{\log_a x = \dfrac{\log_b x}{\log_b a}}$

Example: evaluate $\log_2 9$ with the calculator, using either log_{10} or log_e (i.e. "*ln*")

with base 10	with base e
$\log_2 9 = \dfrac{\log_{10} 9}{\log_{10} 2} = 3.169925 \ldots$	$\log_2 9 = \dfrac{\log_e 9}{\log_e 2} = \dfrac{\ln 9}{\ln 2} = 3.169925 \ldots$

Results are equal indeed, and we can check that: $2^{3.169925} \approx 9$