MATHEMATICAL INDUCTION, HARDER QUESTIONS

Use mathematical induction to prove the following results.

1 Prove that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$ for n a positive integer.

3 Prove that $n^2 + 2n$ is divisible by 8 if n is an even integer.

4 Prove that $3^{4n} - 1$ is divisible by 80 for n a positive integer.

9 Prove that $\sum_{k=1}^{n} \log\left(\frac{k+1}{k}\right) = \log(n+1)$.

11 Prove that $x^n - 1$ is divisible by $(x-1)$ for n a positive integer. Use the result that $\frac{x^n-1}{x-1} = x^{n-1} + \frac{x^{n-1}-1}{x-1}$

12 Prove that $\sum_{r=1}^{n} r \log\left(\frac{r+1}{r}\right) = \log \frac{(n+1)^n}{n!}$.