TRIGONOMETRIC FUNCTIONS AND GRAPHS - CHAPTER REVIEW

1 Solve the trigonometric equations. Round answers to the nearest minute.

(a) $5\sin 2x = 2\tan 30°$, $0° < x < 180°$

(b) $3\sin x = \tan x$, $0° < x < 360°$

a/ $5\sin 2x = 2\times\frac{\sin 30}{\cos 30} = 2\,\frac{1/2}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$

so $\sin 2x = \frac{2}{5\sqrt{3}} = \sin 13°21'$. so $2x = 13°21' \times (-1)^n + n\pi$

$x = 6°41' \times (-1)^n + n\pi/2$

For $n=0$ $\boxed{x = 6°41'}$ For $n=1$ $\boxed{x = 83°19'}$

b/ $3\sin x = \tan x = \frac{\sin x}{\cos x} \Leftrightarrow \sin x\left[3 - \frac{1}{\cos x}\right] = 0$

So either $\sin x = 0$ i.e $\boxed{x = 0°}$ or $\boxed{360°}$

or $3 - \frac{1}{\cos x} = 0$ i.e. $\cos x = \frac{1}{3} = \cos(70°32')$

so $x = \pm 70°32' + n \times 360$

For $n=0$ $\boxed{x = 70°32'}$

$n=+1$ $\boxed{x = 289°28'}$

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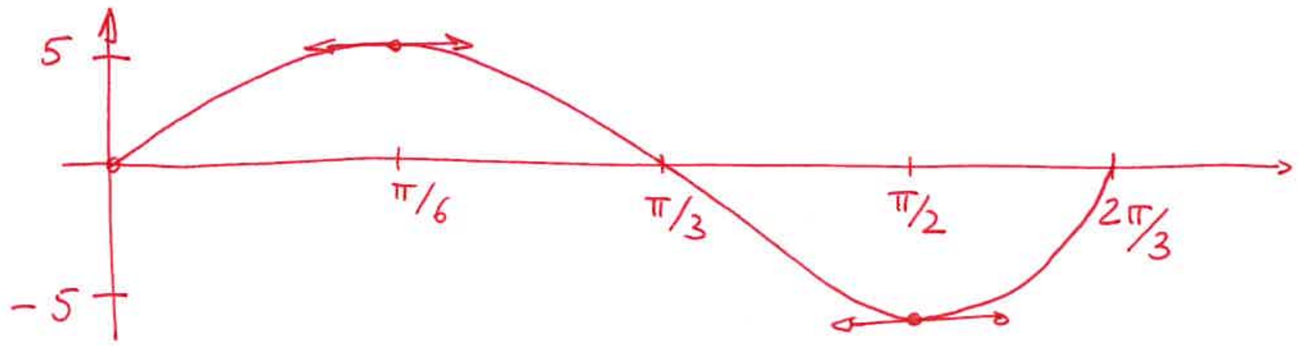
2 Sketch each trigonometric function. Show at least one period. Assume x is in radians.

(a) $y = 5\sin 3x$ **(b)** $y = 2\tan\frac{\pi x}{3} + 4$ **(c)** $y = 4\cos 2\left(x + \frac{\pi}{3}\right)$

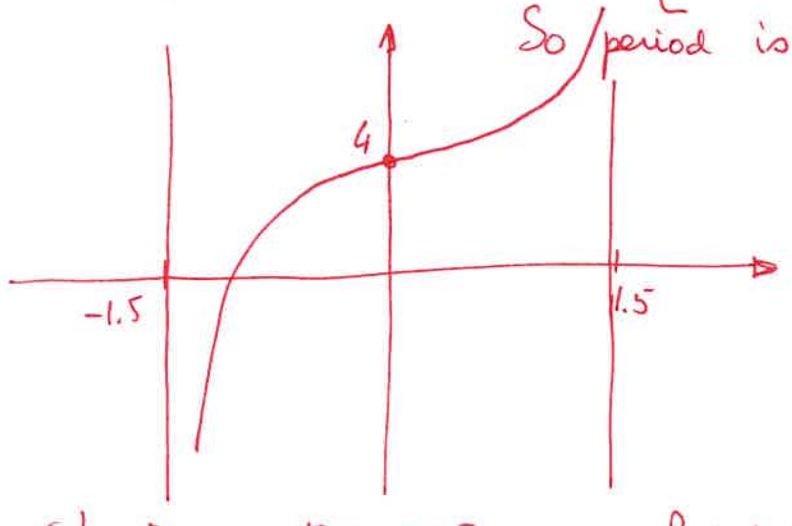
a) Range $[-5, 5]$ ~~Domain~~ Period $\frac{2\pi}{3}$



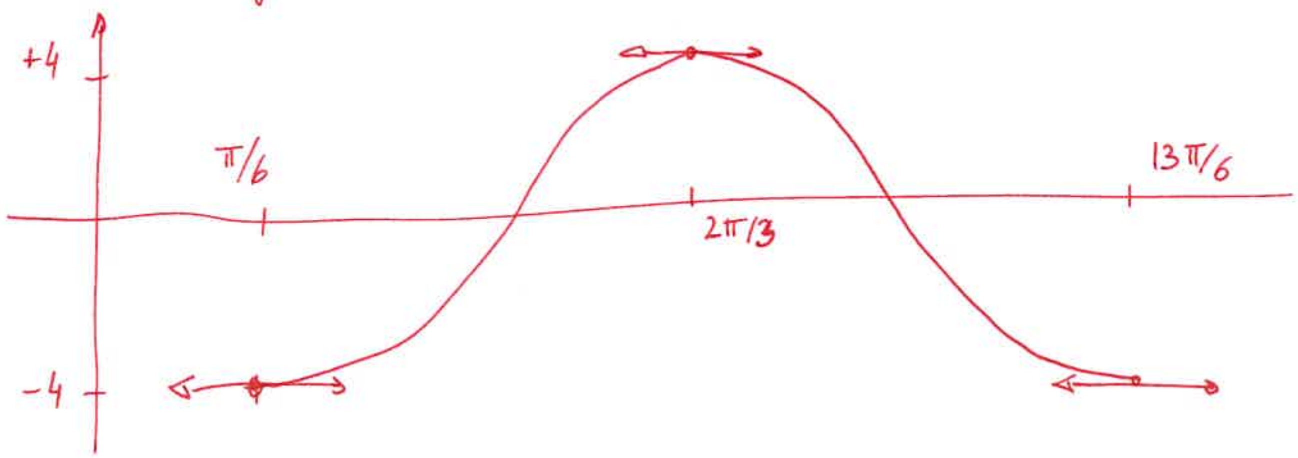
b) Range $(-\infty, +\infty)$ Period $\left[\frac{\pi x}{3} = \frac{\pi}{2} \text{ so } x = \frac{3}{2}\right]$ so $-\frac{3}{2}$ to $\frac{3}{2}$

So period is 3.

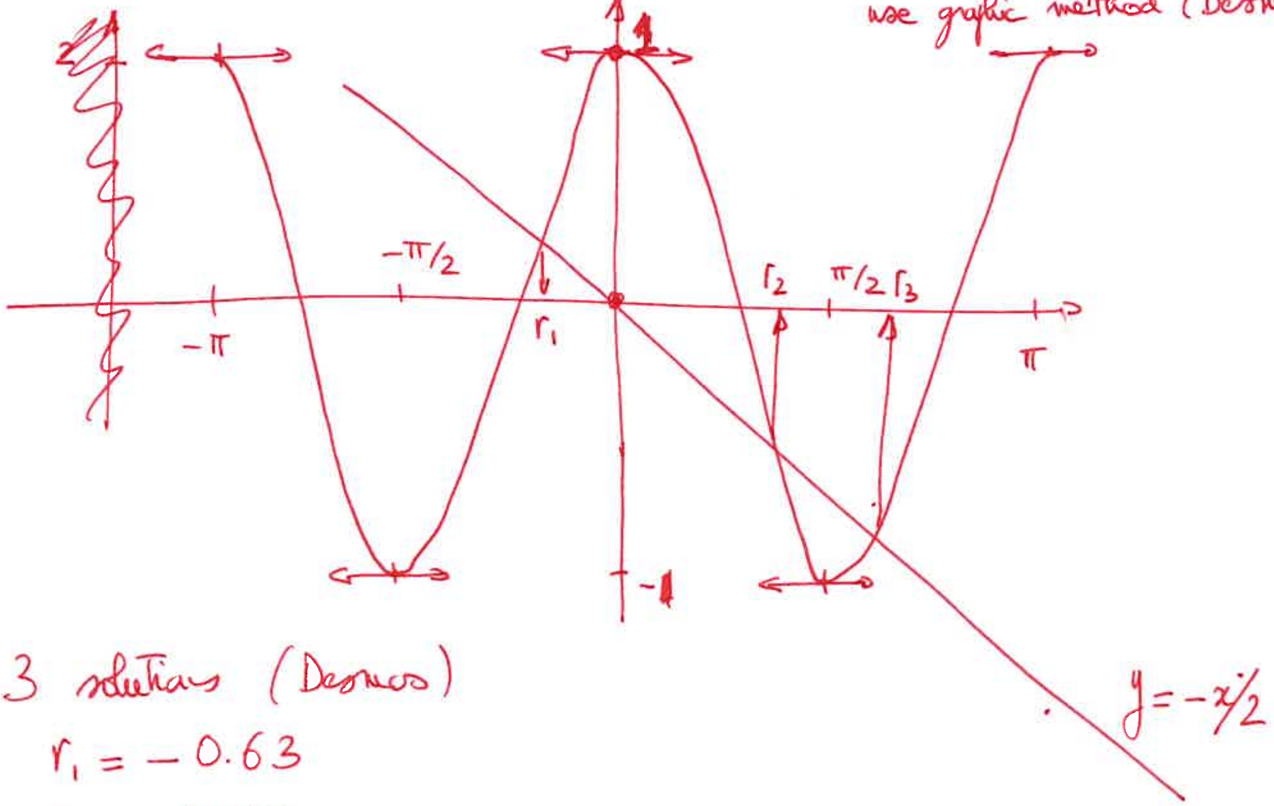
for $x = 0$ $y = 4$



c) Range $[-4, +4]$ Period π



4 Draw the graph of $y = \cos 2x$ for $-\pi \le x \le \pi$. On the same set of axes, draw the graph of $y = -\frac{x}{2}$. Use your graphs to solve the equation $\cos 2x = -\frac{x}{2}$. ← transcendental equation
use graphic method (Desmos)



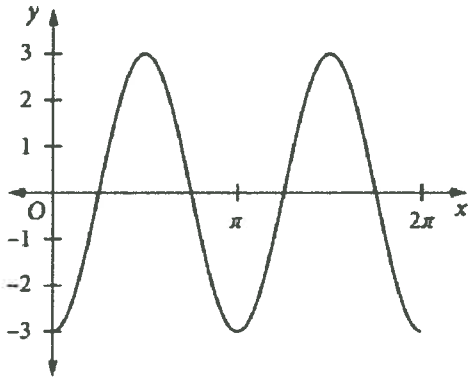
3 solutions (Desmos)

$r_1 = -0.63$

$r_2 = 1.07$

$r_3 = 1.8$

5 State the function represented by the graph, assuming it is a cosine function.



Period is π

Amplitude is 3

Range $[-3, 3]$

So $y = 3\cos(2x + \alpha)$

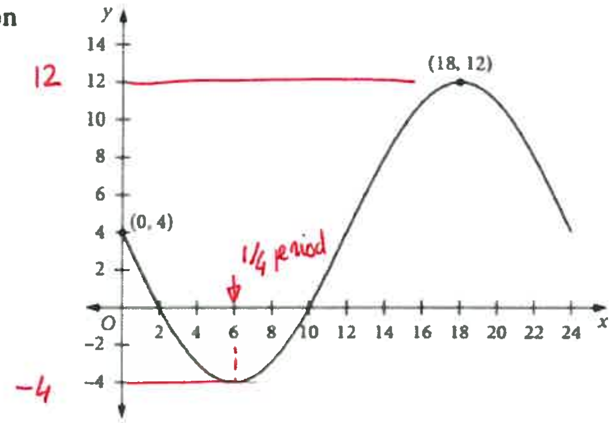
For $x = 0$ $y = -3 = 3\cos\alpha$ so $\cos\alpha = -1$

$\alpha = \pi$

$y = 3\cos(2x + \pi) = -3\cos 2x$

6 The temperature of a pond, in °C, is given by the function $f:[0,24] \to \mathbb{R}, f(x) = k\sin(ax) + c$ where x is time in hours after midnight. $y = f(x)$ is shown.

(a) State the value of c.
(b) State the value of k.
(c) State the period of f.
(d) Find the value of a.
(e) Find the minimum temperature that the pond reaches.
(f) By solving an appropriate equation, find the times at which the temperature of the pond is 0°C over the given domain.



a) $\frac{4+12}{2} = \frac{16}{2} = 8$ so ~~...~~ amplitude is 8 so $c = 8-4 = 4$

b) $k = -8$

c) Period is $4 \times 6 = 24$ (as 6 is $\frac{1}{4}$ of a period)

d) $f(x) = -8\sin(ax) + 4$ $\qquad 24a = 2\pi$ so $a = \frac{\pi}{12}$

$f(x) = -8\sin\left(\frac{\pi x}{12}\right) + 4$

e) When $T = -4°$

f) $f(x) = 0$ when $0 = -8\sin\left(\frac{\pi x}{12}\right) + 4$

$8\sin\left(\frac{\pi x}{12}\right) = 4$ or $\sin\left(\frac{\pi x}{12}\right) = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

so $\frac{\pi}{12}x = \frac{\pi}{6}(-1)^n + n\pi$ $\cancel{x = 2}$ or $x = 2(-1)^n + n \times 12$

$n = 0$ $\boxed{x = 2}$ 2am $\qquad n = 2$ $x = 24 + 2 = 26$ out of period

$n = 1$ $\boxed{x = 10}$ 10 am

9 Solve **(a)** $\cos\left(2x-\frac{\pi}{4}\right)=-1$ for $0\le x\le 2\pi$ **(b)** $\sin\left(2x+\frac{\pi}{3}\right)=0.5$ for $-\pi\le x\le \pi$

a) $\cos\left(2x-\frac{\pi}{4}\right)=\cos\pi$ so $2x-\frac{\pi}{4}=\pm\pi+2n\pi$

$2x=\frac{\pi}{4}\pm\pi+2n\pi$

$x=\frac{\pi}{8}\pm\frac{\pi}{2}+n\pi$

for $n=0$ $x=\frac{\pi}{2}+\frac{\pi}{8}=\frac{5\pi}{8}$

for $n=1$ $x=\frac{13\pi}{8}$ other values are outside of $[0,2\pi]$

b) $\sin\left(2x+\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{6}\right)$

So $2x+\frac{\pi}{3}=(-1)^n\frac{\pi}{6}+n\pi$

$2x=-\frac{\pi}{3}+(-1)^n\frac{\pi}{6}+n\pi$

$x=-\frac{\pi}{6}+(-1)^n\frac{\pi}{12}+n\frac{\pi}{2}$

For $n=0$ $x=-\pi/6+\pi/12=-\pi/12$

for $n=1$ $x=-\pi/6-\pi/12+\pi/2=\pi/4$

for $n=2$ $x=-\pi/6+\pi/12+\pi=11\pi/12$

for $n=-1$ $x=-\pi/6-\pi/12-\pi/2=-3\pi/4$

others solutions are outside of $[-\pi,\pi]$

So $-\frac{3\pi}{4},-\frac{\pi}{12},\frac{\pi}{4},\frac{11\pi}{12}$

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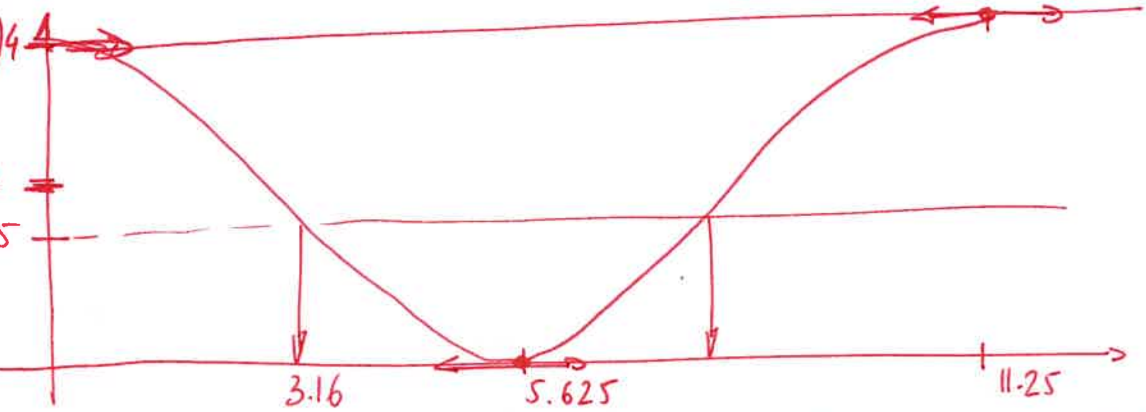
7 Tess likes to swim out to a small island and collect shells at low tide. However, for much of the time this island is submerged. To avoid the disappointment of going to collect shells and finding the island submerged, Tess consults the local marine and tide charts for next Sunday. She discovers that the island is above water when the tide is 1.5 metres above the low water mark. The height, h, in metres of the tide above the low-water mark is given by the equation $h(t) = 2\cos\left(\frac{8\pi}{45}t\right) + 2$, where t is the time in hours after midday Sunday.

(a) Draw a graph showing the height above the low water mark for the 24 hours commencing at midday Sunday.

(b) To the nearest minute, what time will the island first appear above the water on Sunday afternoon?

(c) To the nearest minute, how long does Tess have to collect shells before the island becomes submerged again?

a)



$\frac{8\pi}{45}t = 2\pi$ so $\frac{4t}{45} = 1$ $t = \frac{45}{4}$ is the period

b) 3.16 according to Desmos.

c) about 4 hours 43′ according to Desmos.