INTRODUCTION TO LIMITS

1 Evaluate each limit.

(a) $\lim_{x\to 3}(3x)$

$= 9$

(b) $\lim_{x\to -1}\left(x^2+4x\right)$

$= -3$

(c) $\lim_{x\to 3}\left(9-x^2\right)$

$= 0$

(d) $\lim_{x\to -2}\left(x^2-2x+1\right)$

$= 9$

(e) $\lim_{x\to -4} x^2(x+2)$

$= -32$

(f) $\lim_{h\to 2}\left(h^2-4h+4\right)$

$= 0$

(g) $\lim_{a\to -1}(a+3)(a-4)$

$= -10$

(h) $\lim_{x\to 3}\left(\frac{x^2-5}{x+2}\right)$

$= 4/5$

2 The value of $\lim_{x\to -3}\frac{(x+5)\cancel{(x+3)}}{\cancel{x+3}} = \lim_{x\to -3} x+5$

A -2 **B** 0 $\boxed{\textbf{C}}$ 2 **D** indeterminate

3 Evaluate the following limits.

(a) $\lim_{x\to 3} 1 = 1$

(b) $\lim_{x\to 0}\left(\frac{x^2+5x}{x}\right)$

$= \lim_{x\to 0} x+5$

$= 5$

(c) $\lim_{x\to -2}\left(\frac{x^3+8}{x+2}\right) = \lim_{x\to -2}\frac{(x+2)(x^2-2x+4)}{x+2}$

$= \lim_{x\to -2} x^2-2x+4$

$= 12$

(d) $\lim_{x\to 3}\left(\frac{x^2-5x+6}{x-3}\right)$

$= \lim_{x\to 3}\frac{(x-3)(x-2)}{x-3}$

$= \lim_{x\to 3}(x-2) = 1$

(e) $\lim_{x\to 3}\left(\frac{3x}{x+3}\right)$

$= \frac{9}{6} = \frac{3}{2}$

(f) $\lim_{x\to 5}\left(\frac{x-5}{2x^2-9x-5}\right)$

$= \lim_{x\to 5}\frac{x-5}{(x-5)(2x+1)}$

$= \lim_{x\to 5}\frac{1}{2x+1} = \frac{1}{11}$

(g) $\lim_{x\to 1}\left(\frac{x-1}{x^2+x-2}\right)$

$= \lim_{x\to 1}\frac{x-1}{(x-1)(x+2)}$

$= \lim_{x\to 1}\frac{1}{x+2} = \frac{1}{3}$

(h) $\lim_{x\to 4}\left(\frac{x-1}{x^2+x-2}\right)$

$= \lim_{x\to 4}\frac{x-1}{(x-1)(x+2)}$

$= \lim_{x\to 4}\frac{1}{x+2} = \frac{1}{6}$

(i) $\lim_{x\to 1}\left(\frac{x^3-1}{x-1}\right)$

$= \lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{x-1}$

$= \lim_{x\to 1}(x^2+x+1)$

$= 3$

INTRODUCTION TO LIMITS

4 Evaluate each limit.

(a) $\lim_{x \to 0} f(x)$ where $f(x) = \begin{cases} x^2+1 & \text{for } x \geq 0 \\ 1 & \text{for } x < 0 \end{cases}$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} (x^2+1) = 1$

So the function is continuous over $\mathbb{R}$, as

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

(b) $\lim_{x \to 1} f(x)$ where $f(x) = \begin{cases} 2x & \text{for } x \geq 1 \\ -2x+4 & \text{for } x < 1 \end{cases}$

$\lim_{x \to 1} f(x) = \lim_{x \to 1} 2x = 2$

Note: $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (-2x+4)$

$= 2$

so f is continuous over $\mathbb{R}$ as

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

5 Evaluate $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ where:

(a) $f(x) = x^2 - 1$ **(b)** $f(x) = 2x^2 - 3x + 2$

a) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[(x+h)^2 - 1] - [x^2 - 1]}{h} = \lim_{h \to 0} \frac{[\cancel{x^2} + 2hx + h^2 - \cancel{1} - \cancel{x^2} + \cancel{1}]}{h}$

$= \lim_{h \to 0} \frac{2hx + h^2}{h} = \lim_{h \to 0} (2x + h) = 2x$

b) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[2(x+h)^2 - 3(x+h) + 2] - [2x^2 - 3x + 2]}{h}$

$= \lim_{h \to 0} \frac{[\cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h + \cancel{2} - \cancel{2x^2} + \cancel{3x} - \cancel{2}]}{h}$

$= \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h}$

$= \lim_{h \to 0} 4x + 2h - 3$

$= 4x - 3$

5 Evaluate $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ where:

(c) $f(x) = x^3$ **(d)** $f(x) = x(6-x)$

c) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

$\_\_\_ = \lim_{h \to 0} \frac{[\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}]}{h}$

$\_\_\_ = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$

$\_\_\_ = \lim_{h \to 0} 3x^2 + 3xh + h^2 = 3x^2$

d) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)(6-(x+h)) - x(6-x)}{h}$

$\_\_\_ = \lim_{h \to 0} \frac{\cancel{6x} - \cancel{x^2} - hx + 6h - xh - h^2 - \cancel{6x} + \cancel{x^2}}{h}$

$\_\_\_ = \lim_{h \to 0} \frac{-2xh + 6h - h^2}{h}$

$\_\_\_ = \lim_{h \to 0} -2x + 6 - h$

$\_\_\_ = -2x + 6$

INTRODUCTION TO LIMITS

6 Show that the following limits do not exist:

(a) $\lim_{x\to 0}\frac{1}{x}$	**(b)** $\lim_{x\to 0} f(x)$ where $f(x)=\begin{cases}0 & \text{for } x<0\\ 1 & \text{for } x>0\end{cases}$	**(d)** $\lim_{x\to 0} f(x)$ where $f(x)=\begin{cases}x^2+1 & \text{for } x>0\\ 2 & \text{for } x<0\end{cases}$

a) $\lim_{x\to 0^+}\frac{1}{x} = +\infty$ and $\lim_{x\to 0^-}\frac{1}{x} = -\infty$ so not equal.

b) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 0 = 0$

whereas $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 1 = 1$ so not equal

c) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2 = 2$

whereas $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2+1 = 1$ so not equal.

7 The function whose graph is shown is:

A discontinuous at $x = 0$ Yes.

B continuous for all x NO

C discontinuous at $x = 2$ NO

D continuous for all $x > 0$ but discontinuous at $x = 1$

NO