# VECTORS IN TWO DIMENSIONS

**1** If vector $\underset{\sim}{a}$ is represented by the ordered pair (2, −6), specify an ordered pair for each of the following vectors.

**(a)** $3\underset{\sim}{a}$ **(b)** $\frac{1}{2}\underset{\sim}{a}$ **(c)** $-\underset{\sim}{a}$ **(d)** $0.4\underset{\sim}{a}$

a) $3(2,-6) = (6,-18)$

b) $\frac{1}{2}(2,-6) = (1,-3)$

c) $-(2,-6) = (-2,6)$

d) $0.4 \times (2,-6) = (0.8,-2.4)$

**2** If vector $\underset{\sim}{b}$ is represented by the column vector $\begin{pmatrix}-4\\5\end{pmatrix}$, specify a column vector for each of the following vectors.

**(a)** $-2\underset{\sim}{b}$ **(b)** $5\underset{\sim}{b}$ **(c)** $\frac{1}{3}\underset{\sim}{b}$ **(d)** $-\frac{5}{4}\underset{\sim}{b}$

a) $-2\begin{pmatrix}-4\\5\end{pmatrix} = \begin{pmatrix}8\\-10\end{pmatrix}$

b) $5\begin{pmatrix}-4\\5\end{pmatrix} = \begin{pmatrix}-20\\25\end{pmatrix}$

c) $\frac{1}{3}\begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}4/3\\5/3\end{pmatrix}$

d) $-\frac{5}{4}\begin{pmatrix}-4\\5\end{pmatrix} = \begin{pmatrix}-5\\-\frac{25}{4}\end{pmatrix}$

**4** If $\underset{\sim}{c}$ is the position vector of (6, −3), represent each of the vectors as a column vector.

**(a)** $-\underset{\sim}{c}$ **(b)** $2\underset{\sim}{c}$ **(c)** $-\frac{1}{3}\underset{\sim}{c}$ **(d)** $1.5\underset{\sim}{c}$

a) $-\vec{c} = -\begin{pmatrix}6\\-3\end{pmatrix} = \begin{pmatrix}-6\\3\end{pmatrix}$

b) $2\vec{c} = 2\begin{pmatrix}6\\-3\end{pmatrix} = \begin{pmatrix}12\\-6\end{pmatrix}$

c) $-\frac{1}{3}\vec{c} = -\frac{1}{3}\begin{pmatrix}6\\-3\end{pmatrix} = \begin{pmatrix}-2\\1\end{pmatrix}$

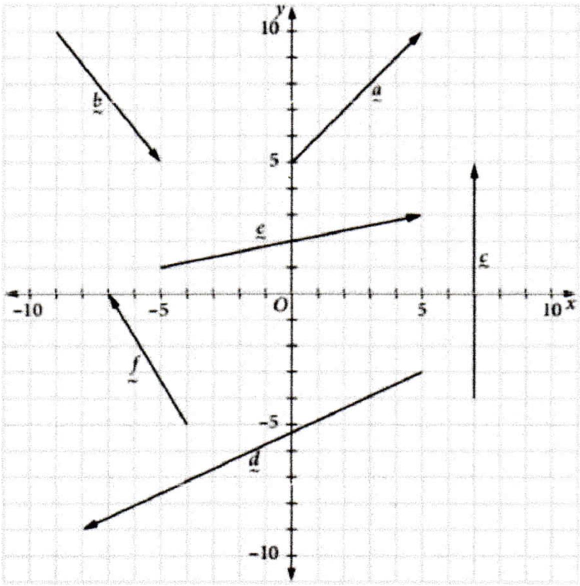
d) $1.5\vec{c} = 1.5\begin{pmatrix}6\\-3\end{pmatrix} = \begin{pmatrix}9\\-4.5\end{pmatrix}$

## VECTORS IN TWO DIMENSIONS

Represent

5 Menu resent each of the vectors in the plane shown as an ordered pair.

(a) $\underset{\sim}{a}$ (b) $\underset{\sim}{b}$ (c) $\underset{\sim}{c}$
(d) $\underset{\sim}{d}$ (e) $\underset{\sim}{e}$ (f) $\underset{\sim}{f}$




a) $\underset{\sim}{a} = (5, 5)$

b) $\underset{\sim}{b} = (4, -5)$

c) $\underset{\sim}{c} = (0, 9)$

d) $\underset{\sim}{d} = (-13, -5)$

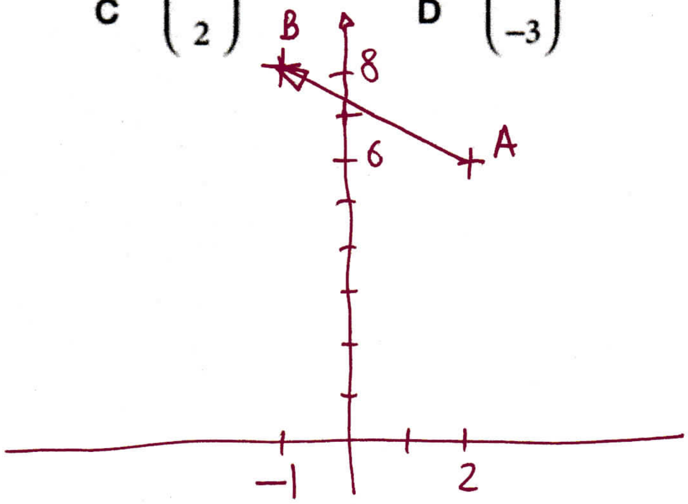
e) $\underset{\sim}{e} = (10, 2)$

f) $\underset{\sim}{f} = (-3, 5)$

A B

8 Which of the following represents the vector from the point (2, 6) to the point (−1, 8)?

A $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ B $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ C $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ D $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

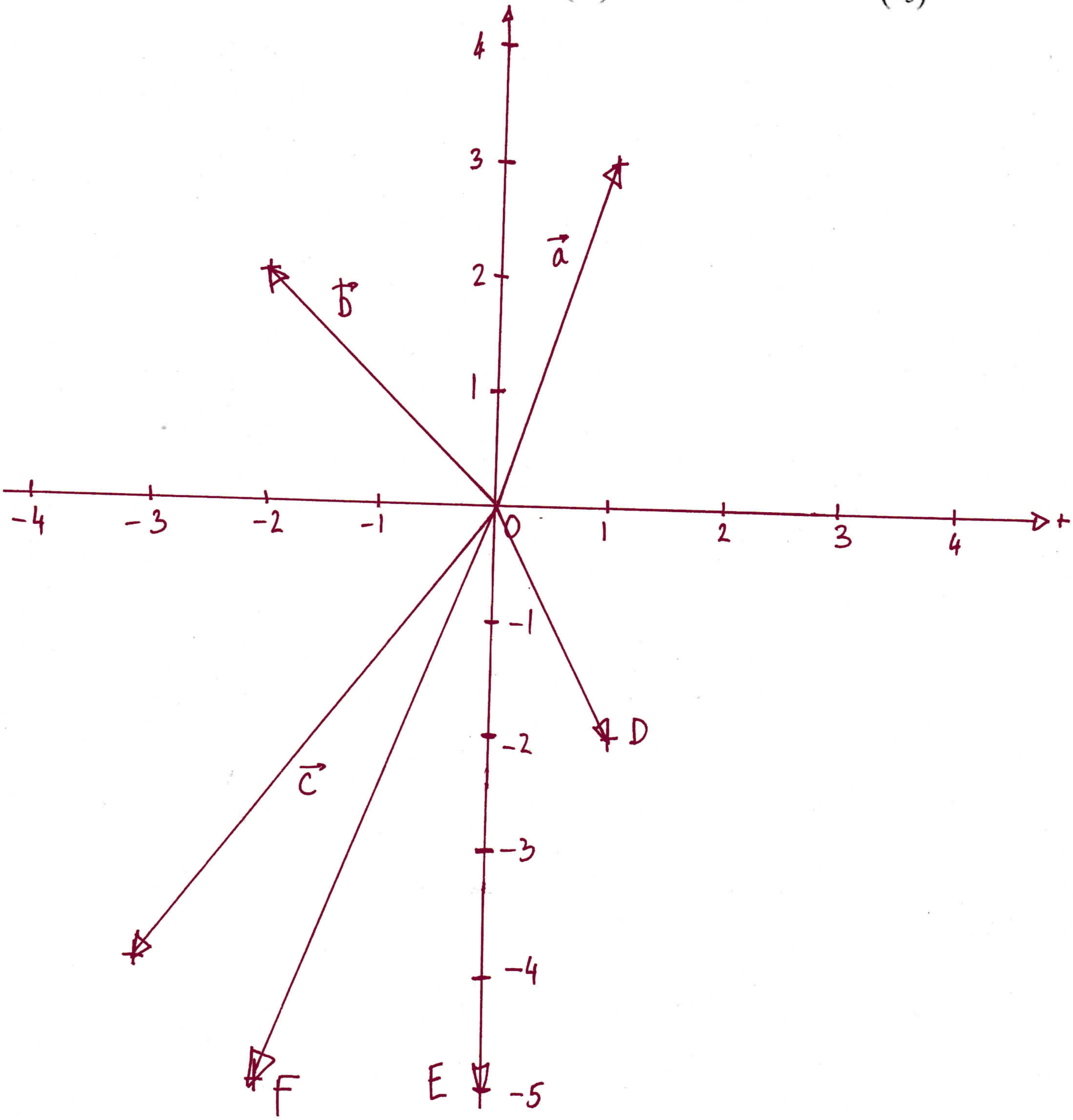



So $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

10 Draw the following vectors on the Cartesian plane.

(a) $\underset{\sim}{a}$, the position vector of $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (b) $\underset{\sim}{b}$, the position vector of $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ (c) $\underset{\sim}{c}$, the position vector of $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

(d) $\overrightarrow{OD}$, where $D$ is $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ (e) $\overrightarrow{OE}$, where $E$ is $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$ (f) $\overrightarrow{OF}$, where $F$ is $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$



**12** The points $A$, $B$ and $C$ have coordinates $(-2, -3)$, $(2, 3)$ and $(8, -1)$ respectively.

**(a)** Find the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ and express them in column vector form.

**(b)** Find $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{AC}|$.

**(c)** Use Pythagoras' theorem to prove that $\Delta ABC$ is a right-angled triangle.

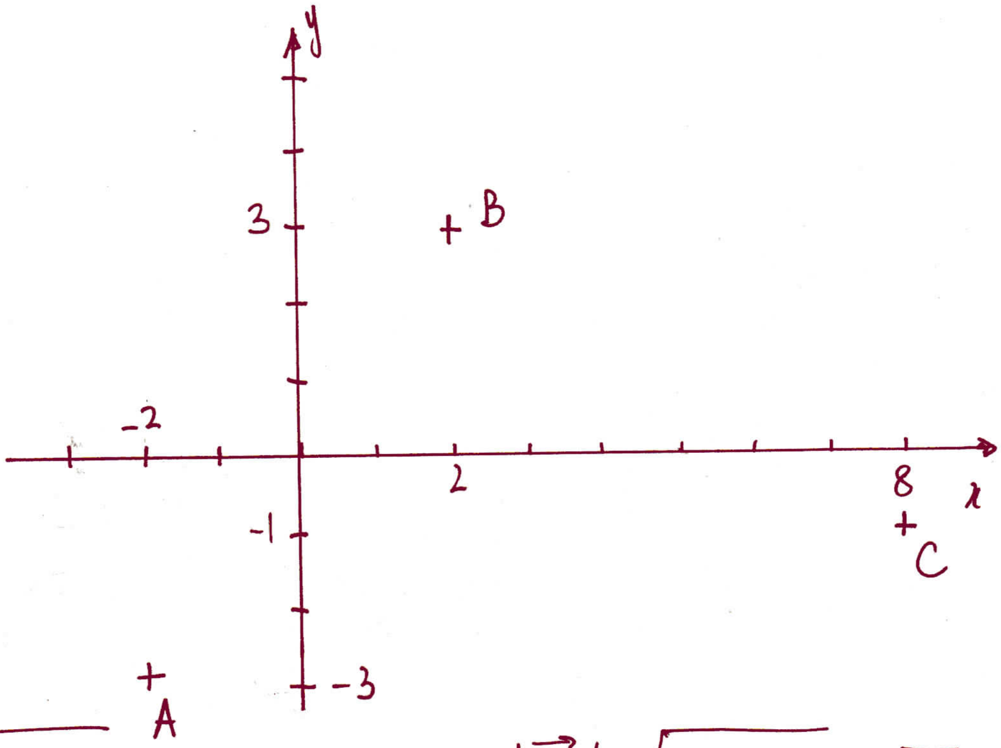
**(d)** Find the coordinates of a point $D$ such that $ABCD$ forms a square.

**(e)** Find the coordinates of the point of intersection of the diagonals of the square $ABCD$.

a/ $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 10 \\ 2 \end{pmatrix}$




b) $|\overrightarrow{AB}| = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$ $\quad |\overrightarrow{BC}| = \sqrt{6^2 + 4^2} = 2\sqrt{13}$

$|\overrightarrow{AC}| = \sqrt{100 + 4} = \sqrt{104} = 2\sqrt{26}$

c) $(2\sqrt{26})^2 = 104$ whereas $(2\sqrt{13})^2 + (2\sqrt{13})^2 = 2 \times 52 = 104$

so $\Delta ABC$ ~~rectangle~~ right-angled

d/ First $|\overrightarrow{AB}| = |\overrightarrow{BC}|$ so 2 sides of the rectangle are equal.

$\overrightarrow{AD} = \overrightarrow{BC}$ so $\overrightarrow{AD} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ so $D((6-2), (-4-3))$

$D(4, -7)$

e/ We call this point of intersection E

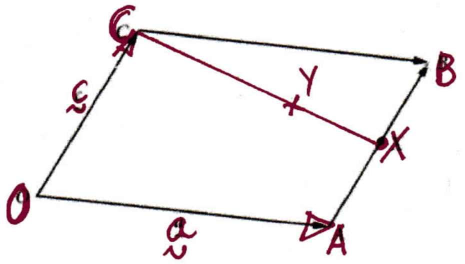
$\overrightarrow{AE} = \frac{1}{2} \overrightarrow{AC} = \frac{1}{2} \begin{pmatrix} 10 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ so as $A(-2, -3)$



then $E(3, -2)$

# VECTORS IN TWO DIMENSIONS

**13** $OABC$ is a parallelogram with $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. $X$ is the midpoint of $\overrightarrow{AB}$ as shown.




**(a)** Find the vectors $\overrightarrow{OB}$ and $\overrightarrow{OX}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

**(b)** Find the vector $\overrightarrow{CX}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

**(c)** If $Y$ is a point on $\overrightarrow{CX}$, such that $\overrightarrow{CY} = \frac{2}{3}\overrightarrow{CX}$, find $\overrightarrow{CY}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

**(d)** Find $\overrightarrow{OY}$ and hence show that $Y$ lies on $\overrightarrow{OB}$.

**(e)** Find the ratio $\overline{OY} : \overline{YB}$.

a) $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \vec{a} + \vec{c}$

$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX} = \vec{a} + \frac{1}{2}\vec{c}$

b) $\overrightarrow{CX} = \overrightarrow{CO} + \overrightarrow{OX} = -\vec{c} + \vec{a} + \frac{1}{2}\vec{c} = \vec{a} - \frac{1}{2}\vec{c}$

c) $\overrightarrow{CY} = \frac{2}{3}\overrightarrow{CX} = \frac{2}{3}\left[\vec{a} - \frac{1}{2}\vec{c}\right] = \frac{2}{3}\vec{a} - \frac{1}{3}\vec{c}$

d) $\overrightarrow{OY} = \overrightarrow{OC} + \overrightarrow{CY}$

So $\overrightarrow{OY} = \vec{c} + \left(\frac{2}{3}\vec{a} - \frac{1}{3}\vec{c}\right) = \frac{2}{3}\vec{c} + \frac{2}{3}\vec{a} = \frac{2}{3}\left(\vec{a} + \vec{c}\right)$

So $\overrightarrow{OY} = \frac{2}{3}\overrightarrow{OB}$

∴ the vectors $\overrightarrow{OY}$ and $\overrightarrow{OB}$ are parallel. As they have one point in common, that means that the points $O, Y$ and $B$ must be aligned.

e) $\frac{\overline{OY}}{\overline{YB}} = \frac{|\overrightarrow{OY}|}{|\overrightarrow{YB}|} = \frac{2/3\,|\overrightarrow{OB}|}{|\overrightarrow{OB}| - |\overrightarrow{OY}|}$ as $|\overrightarrow{YB}| + |\overrightarrow{OY}| = |\overrightarrow{OB}|$ (as the 3 points are aligned)

So $\frac{|\overrightarrow{OY}|}{|\overrightarrow{YB}|} = \frac{2/3\,|\overrightarrow{OB}|}{|\overrightarrow{OB}| - \frac{2}{3}|\overrightarrow{OB}|} = \frac{2/3}{1 - 2/3} = \frac{2/3}{1/3} = \frac{2}{1}$



So $\frac{|\overrightarrow{OY}|}{|\overrightarrow{YB}|} = 2$ $\quad |\overrightarrow{OY}| : |\overrightarrow{YB}| = 2 : 1$

**14 a)** The coordinates of points A and B are respectively $(-3, 6)$ and $(2, 9)$. Find the position vector of the midpoint M of $\overline{AB}$.

**b)** The coordinates of points C and D are respectively $(1, 7)$ and $(5, 3)$. Find the position vectors of the points $P_1$ and $P_2$ trisecting $\overline{CD}$ in three equal parts.

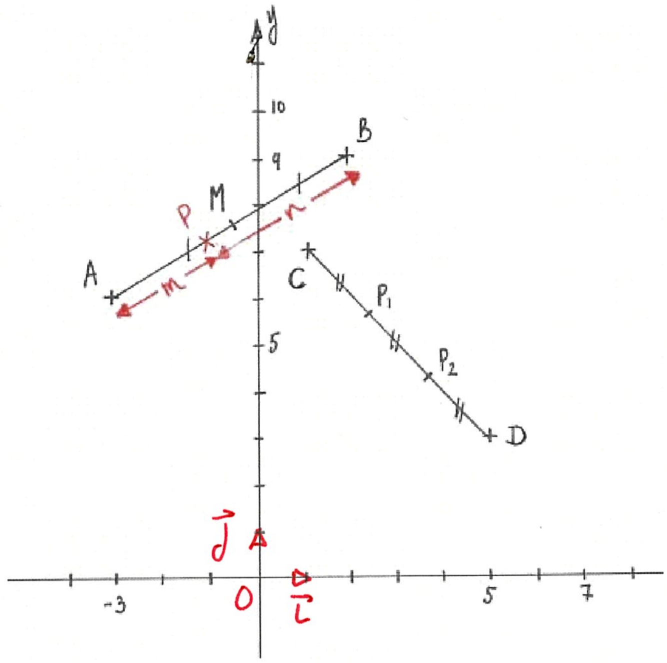
**c)** Let the position vectors of A and B be respectively $\vec{a}$ and $\vec{b}$. Let P be a point which divides $\overline{AB}$ in the ratio $m:n$, so that $\frac{\overline{AP}}{\overline{AB}} = \frac{m}{m+n}$. Show that the position vector of point P is $\frac{n}{m+n}\vec{a} + \frac{m}{m+n}\vec{b}$.

a) The coordinates of the midpoint of $\overline{AB}$ are

$$\left(\frac{-3+2}{2}, \frac{6+9}{2}\right)$$

i.e. $\left(-\frac{1}{2}, \frac{15}{2}\right)$ $\therefore$ $\vec{OM} = -\frac{1}{2}\vec{i} + \frac{15}{2}\vec{j}$

or $\vec{OM}\begin{pmatrix}-1/2\\ 15/2\end{pmatrix}$




b) $\vec{OP_1} = \vec{OC} + \vec{CP_1} = \vec{OC} + \frac{1}{3}\vec{CD} = \vec{i} + 7\vec{j} + \frac{1}{3}\left[4\vec{i} - 4\vec{j}\right] = \frac{7}{3}\vec{i} + \frac{17}{3}\vec{j}$

$\vec{OP_2} = \vec{OC} + \vec{CP_2} = \left[\vec{i} + 7\vec{j}\right] + \frac{2}{3}\vec{CD} = \left[\vec{i} + 7\vec{j}\right] + \frac{2}{3}\left[4\vec{i} - 4\vec{j}\right] = \frac{11}{3}\vec{i} + \frac{13}{3}\vec{j}$

c) $\vec{OP} = \vec{OA} + \vec{AP} = \vec{a} + \frac{m}{m+n}\vec{AB}$

So $\vec{OP} = \vec{a} + \frac{m}{m+n}\left[\vec{AO} + \vec{OB}\right]$

$\vec{OP} = \vec{a} + \frac{m}{m+n}\left[-\vec{a} + \vec{b}\right]$

$\vec{OP} = \vec{a}\left[1 - \frac{m}{m+n}\right] + \left[\frac{m}{m+n}\right]\vec{b} = \frac{n}{m+n}\vec{a} + \frac{m}{m+n}\vec{b}$