Find the missing probability. $P(A \cap B) = P(A) \times P(B|A)$ or $P(A \cap B) = P(B) \times P(A|B)$

19) $P(B) = \frac{1}{2}$ $P(A|B) = \frac{63}{100}$ $P(A \text{ and } B) = ?$

$\frac{1}{2} \times \frac{63}{100} = \frac{63}{200}$

20) $P(A) = \frac{9}{20}$ $P(A \text{ and } B) = \frac{117}{400}$ $P(B|A) = ?$

$P(B|A) = \frac{117/400}{9/20} = 0.65$

1 A box contains 5 black cubes and 3 red cubes. Two cubes are drawn at random from the box. Find the probability that:

(a) both cubes are black **(b)** both cubes are the same colour

(c) both cubes are different colours.

a) $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$

b) either both black or both red

both black: $5/14$

both red: $3/8 \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$

Sum is $13/28$

c) $\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7} = \frac{30}{56} = \frac{15}{28}$ or $1 - \frac{13}{28} = \frac{15}{28}$

3 An angler has caught 15 fish, of which 3 are undersized. A random sample of 3 fish is taken without replacement by an inspector. The angler is fined if one or more of the fish in the sample is undersized. What is the probability that the angler is fined?

P(at least one fish is undersize) + P(none undersize) = 1

So P(at least one fish is undersize) $= 1 - P(\text{none undersize})$

$= 1 - \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13} = \frac{47}{91}$

4 A carton contains a dozen eggs, 3 of which have a double yolk. If 3 eggs are taken to make a cake, find the probability that all 3 eggs will have double yolks.

P(1st egg has double yolk) $= \frac{3}{12} = \frac{1}{4}$

P(3 eggs have double yolks) $= \frac{3}{12} \times \frac{2}{11} \times \frac{1}{10} = \frac{6}{1320} = \frac{1}{220}$

6 From a standard deck of 52 playing cards, two cards are selected at random without replacement. Indicate whether each statement below is correct or incorrect.

(a) $P(\text{both cards are diamonds}) = \frac{1}{17}$

(b) $P(\text{both cards are the same suit}) = \frac{1}{17}$

(c) $P(\text{one card is a spade and the other is a club}) = \frac{13}{102}$

(d) $P(\text{both cards are different suits}) = \frac{13}{17}$

a) $\frac{1}{4} \times \frac{12}{51} = \frac{1}{17}$ correct

b) $\left[\frac{1}{4} \times \frac{12}{51}\right] \times 4 = \frac{12}{51}$ incorrect

c) $\frac{1}{4} \times \frac{13}{51} + \frac{1}{4} \times \frac{13}{51} = \frac{13}{102}$ correct

d) $\left[\frac{1}{4} \times \frac{39}{51}\right] \times 4 = \frac{13}{17}$ correct

11 In a raffle, 20 tickets are sold and there are 2 prizes. If you buy 5 tickets, what is the probability that you win at least one of the prizes?

P(winning at least one prize) + P(winning no price) = 1

$P(\text{NOT winning}) = \frac{18}{20} \times \frac{17}{19} \times \frac{16}{18} \times \frac{15}{17} \times \frac{14}{16} = \frac{21}{38}$ so

$P(\text{winning at least 1 prize}) = 1 - \frac{21}{38} = \frac{17}{38}$

12 Group *A* contains 10 gorillas and 5 chimpanzees. Group *B* contains 4 gorillas and 6 chimpanzees. Two apes are selected at random from the groups. What is the probability that:

(a) they are both gorillas, if they are selected from Group *A*
(b) they are both chimpanzees, if they are selected from Group *B*
(c) one is a gorilla, the other is a chimpanzee, if one is selected from each group?

a) $\frac{10}{15} \times \frac{9}{14} = \frac{3}{7}$ b) $\frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$

c) $\frac{10}{15} \times \frac{6}{10} + \frac{5}{15} \times \frac{4}{10} = \frac{80}{150} = \frac{8}{15}$

13 A box of 10 chocolates contains 4 hard-centred and 6 soft-centred chocolates. If two chocolates are selected at random, what is the probability that:

(a) they both have hard centres **(b)** they both have soft centres
(c) one has a soft centre and the other has a hard centre?

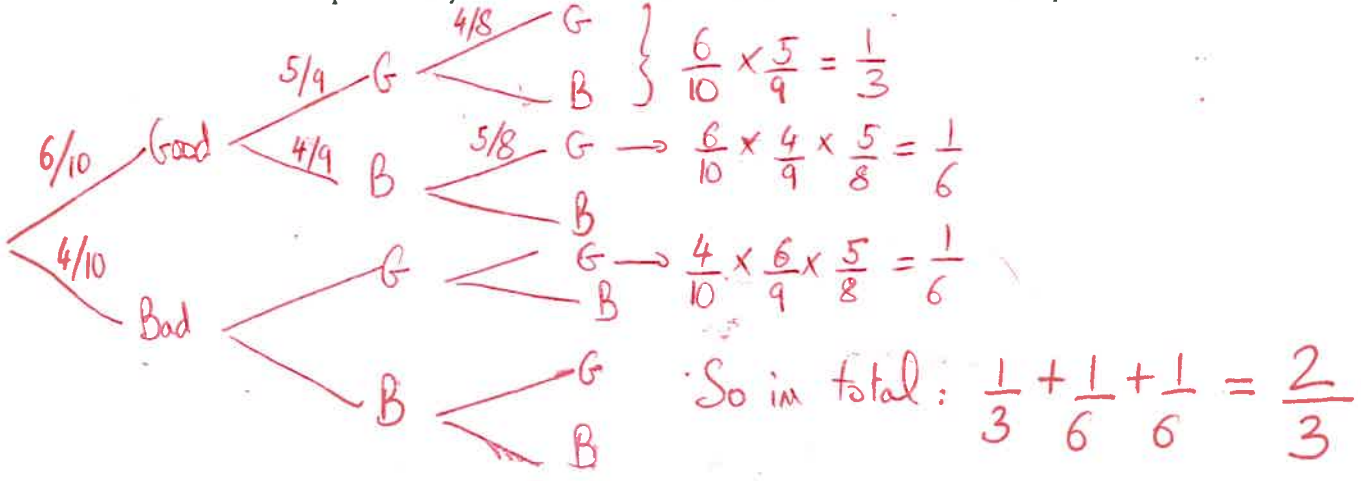
a) $\frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$ b) $\frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$

c) $1 - \left(\frac{2}{15} + \frac{1}{3}\right) = \frac{8}{15}$

14 In a lottery game, three numbers are selected at random from 1, 2, 3, 4, ... 40. Find the probability that the three selected numbers are all even.

$\frac{20}{40} \times \frac{19}{39} \times \frac{18}{38} = \frac{3}{26}$

15 A sample of three items is selected at random without replacement from a batch of ten items, four of which are defective. Find the probability that there is at most one defective item in the sample.



25 On average, at a particular beach it rains on 2 days out of every 7. Find the probability that on a given weekend it will rain:

(a) on both days

$\frac{2}{7} \times \frac{2}{7} = \frac{4}{49}$

(b) on at least one day.

P(rain on at least one day) = 1 − P(no rain)

$= 1 - \left(\frac{5}{7}\right) \times \left(\frac{5}{7}\right)$

$= 1 - \frac{25}{49}$

$= \frac{24}{49}$

27 In a large flock of birds, 50% are red and 50% are green. If two birds are selected at random, what is the probability that:

(a) they are both red

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(b) at least one is green?

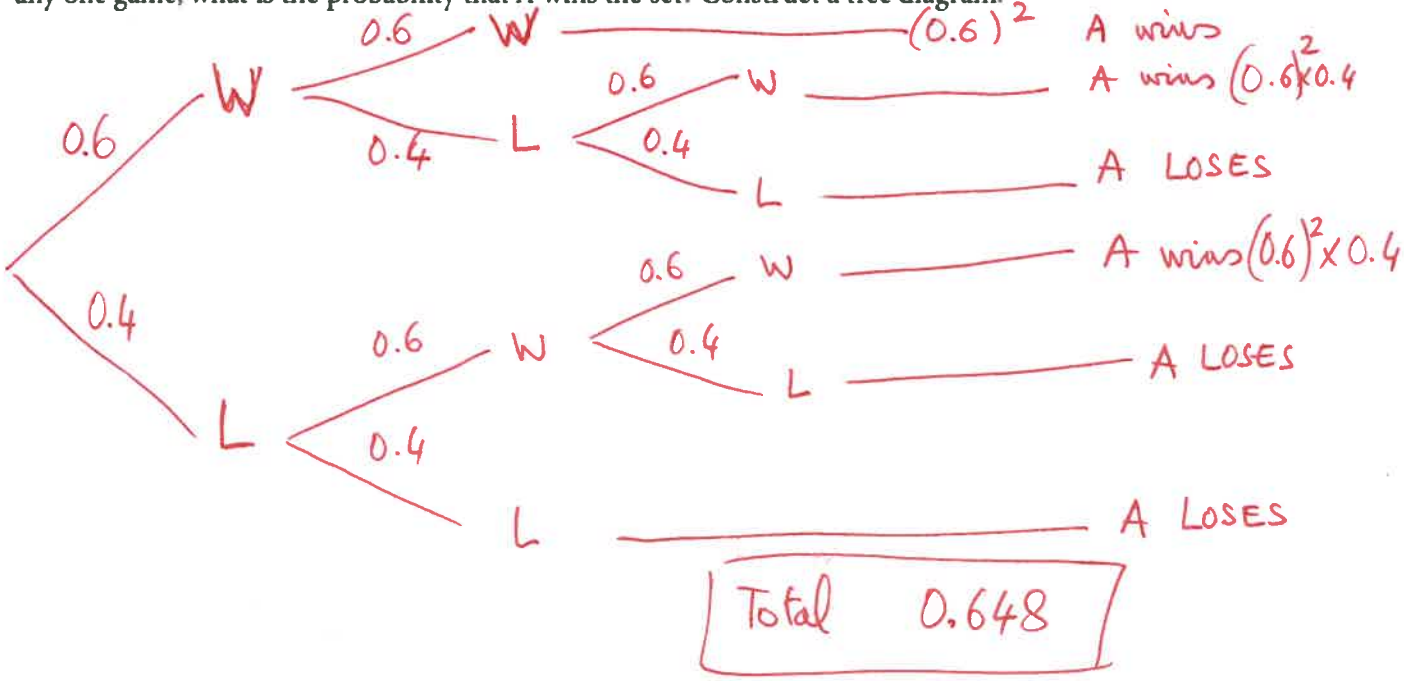
P(at least one green) = 1 − P(none green)

= 1 − P(both red)

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

29 *A* and *B* play a 'set' of tennis: when a player wins two games, the set is won. If *A* has probability 0.6 of winning any one game, what is the probability that *A* wins the set? Construct a tree diagram.



What is the probability of getting at least one head in the tossing of a coin 3 times?

$P(\text{at least 1 head}) = 1 - P(\text{no head}) = 1 - \left(\frac{1}{2}\right)^3 = \frac{7}{8}$

1) Assume two fair dice are rolled. → 36 possibilities

a) What is the probability of rolling a sum of 7 or 11?

7 is either (1,6) (2,5) (3,4) (4,3) (5,2) or (6,1) → 6

11 is either (5,6) or (6,5) → 2

so $\frac{8}{36} = \frac{2}{9}$

b) What is the probability that the sum is a prime number? (2, 3, 5, 7, 11)

$P(2) = 1/36 \quad P(3) = 2/36 \quad P(5) = \frac{1+1+1+1}{36} = \frac{4}{36}$

So $\frac{1}{36} + \frac{2}{36} + \frac{4}{36} + \frac{2}{9} = \frac{15}{36} = \frac{5}{12}$

c) What is the probability that the sum is greater than 7 if you already know that the number showing on one die is 3? Next number can be 5 or 6.

So $\frac{2}{6} = \frac{1}{3}$

d) What is the probability that the sum is at least 7? 7, 8, 9, 10, 11, 12

$\frac{6}{36} + \frac{1+1+1+1+1+1}{36} + \frac{1+1+1+1+1}{36} + \frac{1+1+1+1}{36} + \frac{1+1}{36} + \frac{1}{36} = \frac{21}{36} = \frac{7}{12}$

e) What is the probability that the sum is even and is greater than 7? 8, 10, 12

$\frac{5}{36} + \frac{3}{36} + \frac{1}{36} = \frac{9}{36} = \frac{1}{4}$

(2,6); (3,5), (4,4), (5,3), (6,2)
(4,6), (6,4), (5,5)
(6,6)

2) A true/false test has 10 items. How many different answer keys are possible? 2^{10}

a) If picking answers randomly, what is the probability of getting a 100% score?

$\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$

b) If picking answers randomly, what is the probability of getting at least a 90% score?

$P(\text{at least 90\% score}) = P(90\%) + P(100\%)$

$= \frac{1}{2^9} \times 10 + \frac{1}{2^{10}} = \frac{5}{512}$

10 because either the 1st one is wrong, or the 2nd one, or the 3rd, etc