MATHEMATICAL INDUCTION INVOLVING SERIES

1 If $S(n)$ is the statement that $n + 2n + 3n + \ldots + n^2 = \frac{n^2(n+1)}{2}$, then $S(5)$ represents the statement:

A $1 + 2 + 3 + \ldots + 5 = \frac{5\times6}{2}$

B $1 + 2 + 3 + \ldots + 25 = \frac{25\times26}{2}$

C $1 + 2 + 3 + \ldots + 25 = \frac{25\times6}{2}$

D $5 + 10 + 15 + 20 + 25 = \frac{25\times6}{2}$

Prove each of the following by induction for all positive integers n.

3 $1 + 2 + 4 + \ldots + 2^{n-1} = 2^n - 1$

4 $2 + 5 + 8 + \ldots + (3n - 1) = \frac{n(3n+1)}{2}$

7 $1 + r + r^2 + r^3 + \ldots + r^{n-1} = \dfrac{1 - r^n}{1 - r}$

8 $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

11 $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

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15 **(a)** $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$. Hence find $\lim_{n \to \infty} \left(\frac{1^3 + 2^3 + 3^3 + \ldots + n^3}{n^4} \right)$

(b) Hence show that $1^3 + 2^3 + 3^3 + \ldots + n^3 = (1 + 2 + 3 + \ldots + n)^2$.

16 $(n+1)+(n+2)+\ldots+2n=\dfrac{n(3n+1)}{2}$

17 $1 \times 1! + 2 \times 2! + 3 \times 3! + \ldots + n \times n! = (n+1)! - 1$