GRAPHICAL SOLUTION OF TRIGONOMETRIC EQUATIONS

1 Using Desmos, graph the following functions, and use these graphs to find the number of solutions in $0 \le x \le \pi$

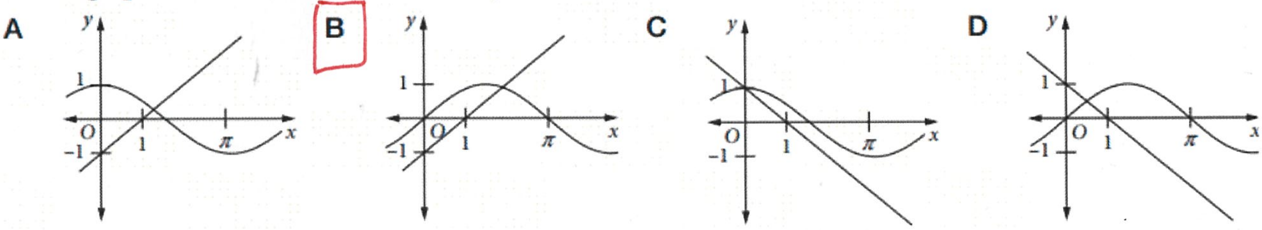
(a) $\sin x = \frac{x}{2}$ (b) $\sin x = \frac{x}{2} - 1$ (c) $\sin x = \frac{x}{2} - x$

a) 2 solutions $x = 0$ and $x \simeq 1.9$

b) 1 solution $x \simeq 2.76$

c) No solutions in $[0, \pi]$

4 Which graph could be used to solve the equation $\sin x = x - 1$?



5 By drawing appropriate graphs using Desmos, determine the number of solutions to the equation $\sin 2x = 1 - \frac{x}{4}$ in the domain $0 \le x \le 2\pi$

5 solutions $x_1 \simeq 0.52$

$x_2 \simeq 1.17$

$x_3 \simeq 3.23$

$x_4 \simeq 4.81$

$x_5 \simeq 6.03$

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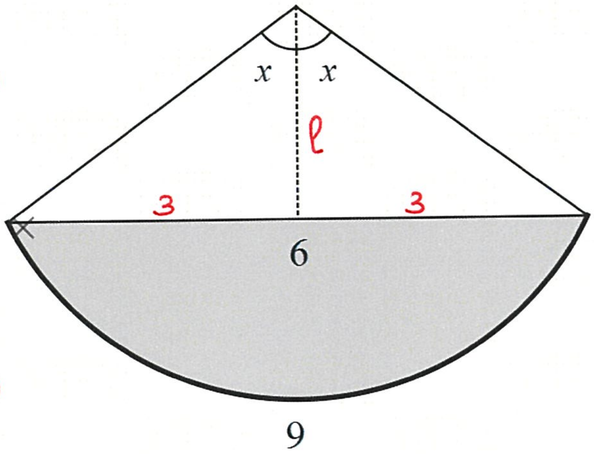
6 A rectangular strip of metal 9 cm wide is bent to form a water channel. The section perpendicular to the length is a circular arc whose chord is 6 cm long.

(a) Show that if the circular arc subtends an angle of $2x$ radians at the centre of the circle, then $3\,sin\,x = 2x$



$\sin x = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{3}{r}$ Equation ①

Also $l = r\theta$ for the length of an arc,

$\therefore\ 9 = r \times 2x$ Equation ②

From ① $\Rightarrow$ $r = \dfrac{3}{\sin x}$ which we substitute in Eq. ②

resulting in $9 = \dfrac{3}{\sin x} \times 2x$

$\Leftrightarrow\ 9 \sin x = 6x$

$\Leftrightarrow\ 3 \sin x = 2x$

(b) Solve the equation in part (a) graphically, giving your answer correct to 1 decimal place.

Graphically, using Desmos, we obtain $x \simeq 1.44$ radians $(\simeq 82.5°)$

(c) Find the area of the cross section of the channel in cm^2 (shaded grey on the figure)

Area of cross section $= \frac{1}{2} r^2 \times (2x) - 3 \times l$ (as $A_{\text{sector}} = \frac{1}{2} r^2 \theta$)

But $\tan x = \dfrac{3}{l}$ so $l = \dfrac{3}{\tan x}$

Area of cross section $= \dfrac{1}{2}\left(\dfrac{9}{2x}\right)^2 \times 2x - \dfrac{9}{\tan x} = \dfrac{81}{4x} - \dfrac{9}{\tan x}$

Area cross section $= \dfrac{81}{4 \times 1.44} - \dfrac{9}{\tan 1.44} \simeq 12.9$ cm^2

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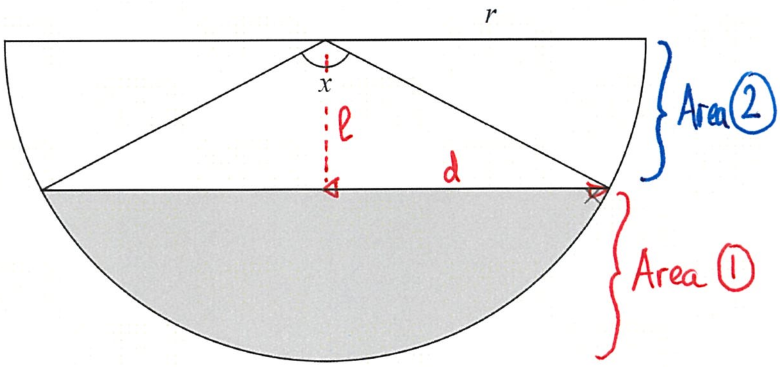
7 A semicircle of radius r is divided into two parts of equal area by a chord parallel to the base (diameter).

(a) If the chord subtends an angle of x radians at the centre, prove that $x - \frac{\pi}{2} = \sin x$

(Note: you will need to use the formula $\sin\frac{x}{2} \times \cos\frac{x}{2} = \frac{\sin x}{2}$)



The area of a sector is given by $\frac{1}{2}r^2\theta$, therefore:

$$\text{Area (1)} = \frac{1}{2}r^2x - 2\times\left(\frac{1}{2}dl\right)$$

$$\text{Area (1)} = \frac{1}{2}r^2x - dl$$

But $\sin\frac{x}{2} = \frac{d}{r}$, so $d = r \times \sin\frac{x}{2}$

and $\cos\frac{x}{2} = \frac{l}{r}$, so $l = r \times \cos\frac{x}{2}$

Therefore: $\text{Area (1)} = \frac{1}{2}r^2x - r\sin\frac{x}{2} \times r\cos\frac{x}{2}$

$\text{Area (1)} = \frac{1}{2}r^2x - r^2\frac{\sin x}{2}$ (using $\sin\frac{x}{2}\cos\frac{x}{2} = \frac{\sin x}{2}$)

$\text{Area (1)} = \frac{r^2}{2}\left[x - \sin x\right]$

We are told that Area (1) = Area (2), i.e. $\frac{\pi r^2}{2} = 2 \times \text{Area (1)}$

Therefore $\frac{\pi r^2}{2} = 2 \times \frac{r^2}{2}\left[x - \sin x\right]$, which simplifies as:

$\pi = 2(x - \sin x)$, or $x - \sin x = \frac{\pi}{2}$

(b) Solve the equation in part (a) graphically, giving your answer correct to 1 decimal place.

Graphically, using Desmos, we obtain $x \simeq 2.3$ rad or $132°$

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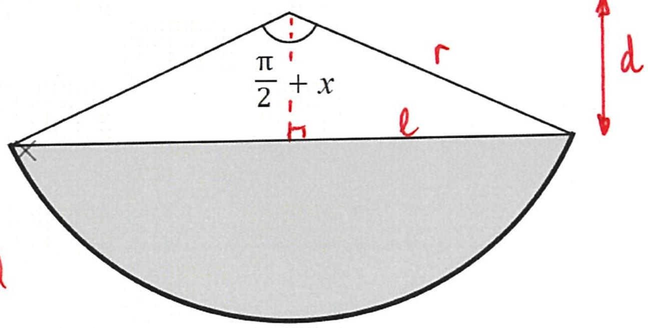
8 The chord of a segment of a circle subtends an angle of $\frac{\pi}{2} + x$ radians at the centre. The area of the segment is one.-quarter the area of the circle.

(a) Prove that $x = \cos x$. Note that you will need to use the formula $\sin\frac{x}{2} \times \cos\frac{x}{2} = \frac{\sin x}{2}$



We are told that:

$$\text{Area}_{\text{segment}} = \frac{1}{4}\pi r^2$$

Also $\underbrace{\text{Area sector}}_{=\frac{1}{2}r^2\left(\frac{\pi}{2}+x\right)} = \text{Area}_{\text{segment}} + 2 \times \frac{1}{2} l d$

$$\therefore \frac{1}{4}\pi r^2 = \frac{1}{2} r^2\left(\frac{\pi}{2}+x\right) - ld \qquad \text{Equation (1)}$$

But $\sin\left[\frac{1}{2}\left(\frac{\pi}{2}+x\right)\right] = \frac{l}{r}$ so $l = r \times \sin\left[\frac{1}{2}\left(\frac{\pi}{2}+x\right)\right]$

and $\cos\left[\frac{1}{2}\left(\frac{\pi}{2}+x\right)\right] = \frac{d}{r}$ so $d = r \times \cos\left[\frac{1}{2}\left(\frac{\pi}{2}+x\right)\right]$

$$\therefore l \times d = r^2 \sin\left[\frac{1}{2}\left(\frac{\pi}{2}+x\right)\right]\cos\left[\frac{1}{2}\left(\frac{\pi}{2}+x\right)\right] = \frac{r^2}{2}\sin\left[\frac{\pi}{2}+x\right] = \frac{r^2}{2}\cos x$$

as $\sin\left(\frac{\pi}{2}+x\right) = \cos x$

$$\text{Equation (1)} \Leftrightarrow \frac{1}{4}\pi r^2 = \frac{1}{2}r^2\left(\frac{\pi}{2}+x\right) - \frac{r^2}{2}\cos x$$

$$\Leftrightarrow \frac{\pi}{4} = \frac{1}{2}\left(\frac{\pi}{2}+x\right) - \frac{1}{2}\cos x$$

$$\Leftrightarrow \pi = 2\left(\frac{\pi}{2}+x\right) - 2\cos x \Leftrightarrow \boxed{x = \cos x}$$

(b) Solve the equation in part (a) graphically, giving your answer correct to 2 decimal places.

Graphically, using Desmos, $x \simeq 0.74$ radians or $42.4°$