1 Use mathematical induction to prove **(c)** $\sum_{r=1}^{n}\left(r^2+1\right)r! = n\times(n+1)!$

2 **(a)** Simplify $\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$. **(b)** Hence evaluate $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$.

(c) Use mathematical induction to prove that $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$ equals the result that you obtained in part **(b)**.

3 Consider the sequence of numbers defined by $T_1 = 3$, $T_n = 2 \times T_{n-1} + 3$ for all $n \geq 2$.

(a) List the first five terms of this sequence.

(b) Prove by induction that $T_n = 3(2^n - 1)$ for all integers $n \geq 1$.

4 **(a)** If $u_{n+1} = 2u_n + 1$ for all positive integral values of n, use mathematical induction to prove that $u_n + 1 = 2^{n-1}(u_1 + 1)$.

6 If $x > 0$ and $y > 0$, prove by induction that $(x + y)^n > x^n + y^n$ for all integers $n \geq 2$.

7 **(a)** By writing $\cos([2k+1]x)$ as $\cos(2kx+x)$, and remembering that $\cos 2x = 1 - 2\sin^2 x$, show that:

$$\frac{\sin 2kx}{2\sin x} + \cos(2k+1)x = \frac{\sin(2(k+1)x)}{2\sin x}$$

(b) Use the result of part **(a)** to prove by induction that $\cos x + \cos 3x + \ldots + \cos([2n-1]x) = \frac{\sin(2nx)}{2\sin x}$ for all positive integers n.

8 If $x_1, x_2, x_3, \ldots x_n$ are positive real numbers, prove by induction that:
$(x_1 + x_2 + x_3 + \ldots + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \ldots + \frac{1}{x_n}\right) \geq n^2$ for all integers $n \geq 1$.

9 **(a)** Prove that $x+\sqrt{x} \geq \sqrt{x(x+1)}$ for all real $x \geq 0$.

(b) A sequence is defined as $u_1 = 1$, $u_2 = 2$, $u_n = u_{n-1} + (n-1)u_{n-2}$ for $n \geq 3$. Prove by induction that $u_n \geq \sqrt{n!}$.