INDEFINITE INTEGRALS

If $F(x)$ is a primitive of $f(x)$, then: $$\int f(x)\,dx = F(x) + C$$

where C is a constant.

$\int f(x)\,dx$ is called the indefinite integral of $f(x)$ and represents all the primitives of $f(x)$. If further information is given, then a value for C may be calculated.

Particularly when $f(x) = x^n$ $(n \neq 1)$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

This formula can be extended to a power of a function, as follows:

$$\int [f(x)]^n\, f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C$$

which is really the chain rule in reverse, as: $\{[f(x)]^{n+1}\}' = (n+1)\,[f(x)]^n\, f'(x)$

Example 6

Find: **(a)** $\int (x^3 + 3x^2 - 2x + 1)\,dx$ **(b)** $\int (\sqrt{x} - 2)\,dx$

Solution

(a) $\int (x^3 + 3x^2 - 2x + 1)\,dx = \frac{x^4}{4} + x^3 - x^2 + x + C$

(b) $\int (\sqrt{x} - 2)\,dx = \int \left(x^{\frac{1}{2}} - 2\right)dx = \frac{2}{3}x^{\frac{3}{2}} - 2x + C$ or $\frac{2x\sqrt{x}}{3} - 2x + C$

Example 7

If $\frac{dy}{dx} = 1 + x - x^2$, find the equation of the curve y that passes through the point $(6, -40)$.

Solution

$\frac{dy}{dx} = 1 + x - x^2$: $\quad y = \int (1 + x - x^2)\,dx$

$$y = x + \frac{x^2}{2} - \frac{x^3}{3} + C$$

Point $(6, -40)$ is on the curve: $\quad -40 = 6 + 18 - 72 + C$

$$C = 8$$

The equation of the curve is $y = x + \frac{x^2}{2} - \frac{x^3}{3} + 8$.

PRIMITIVES OF TRIGONOMETRIC FUNCTIONS

$\frac{d}{dx}(sin\, x) = cos\, x$ therefore $\int cos\, x\, dx = sin\, x + C$

$\frac{d}{dx}(cos\, x) = -sin\, x$ therefore $\int sin\, x\, dx = -cos\, x + C$

$\frac{d}{dx}(tan\, x) = sec^2\, x$ therefore $\int sec^2\, x\, dx = tan\, x + C$

Further, provided $a \neq 0$

$\frac{d}{dx}[sin(ax+b)] = a \times cos(ax+b)$ therefore $\int cos(ax+b)\, dx = \frac{1}{a} sin(ax+b) + C$

$\frac{d}{dx}[cos(ax+b)] = -a \times sin(ax+b)$ therefore $\int sin(ax+b)\, dx = -\frac{1}{a} cos(ax+b) + C$

$\frac{d}{dx}[tan(ax+b)] = a \times sec^2(ax+b)$ therefore $\int sec^2(ax+b)\, dx = \frac{1}{a} tan(ax+b) + C$

Example 8

Find: **(a)** $\int(\sin x + 2\cos x)dx$ **(b)** $\int(\cos 2x + \sin 3x)dx$ **(c)** $\int \sin\frac{x}{2}dx$ **(d)** $\int\left(2\sin x + 3\cos\frac{x}{2}\right)dx$

Solution

(a) $\int(\sin x + 2\cos x)dx$

$= -\cos x + 2\sin x + C$

(b) $\int(\cos 2x + \sin 3x)dx$

$= \frac{1}{2}\sin 2x - \frac{1}{3}\cos 3x + C$

(c) $\int \sin\frac{x}{2}dx$

$= -2\cos\frac{x}{2} + C$

(d) $\int\left(2\sin x + 3\cos\frac{x}{2}\right)dx$

$= -2\cos x + 6\sin\frac{x}{2} + C$