FACTORISING NON-MONIC TRINOMIALS

When the x^2 term in the quadratic has a coefficient over than 1, finding the factors becomes more difficult because there are more possibilities.

You must find factors of the coefficient of x^2 as well as the factors of the constant term and get the correct pairs together.

You could use trial and error but the cross method is easier because it keeps the information more organised.

Example 8

Factorise $6x^2 + 19x + 10$.

Solution

Write: $6x^2 + 19x + 10 = (ax + m)(bx + n) = abx^2 + (an + bm)x + mn$

This gives: $ab = 6$, $an + bm = 19$, $mn = 10$

List the factors of 6: 6, 1 or 3, 2

List the factors of 10: 10, 1 or 5, 2

List possible binomial factors:

$(6x + 10)(x + 1)$ $(6x + 1)(x + 10)$ $(6x + 5)(x + 2)$ $(6x + 2)(x + 5)$

$(3x + 10)(2x + 1)$ $(3x + 1)(2x + 10)$ $(3x + 5)(2x + 2)$ $(3x + 2)(2x + 5)$

You can expand the binomial factors to see which answer gives the original quadratic trinomial.

Before doing this, you can eliminate any possibility that has a common factor, because there is no common factor in the original quadratic trinomial. This means you can eliminate the answers containing $(6x + 10)$, $(6x + 2)$, $(2x + 10)$ and $(2x + 2)$, because they all have a common factor of 2 (and the original quadratic trinomial does not).

Expand the others: $(6x + 1)(x + 10) = 6x^2 + 61x + 10$ Not correct

$(6x + 5)(x + 2) = 6x^2 + 17x + 10$ Not correct

$(3x + 10)(2x + 1) = 6x^2 + 23x + 10$ Not correct

$(3x + 2)(2x + 5) = 6x^2 + 19x + 10$ Correct

Hence: $6x^2 + 19x + 10 = (3x + 2)(2x + 5)$

Alternatively, using the cross method:

$6x$	X	10	1	5	2
x		1	10	2	5

or

$3x$	X	10	1	5	2
$2x$		1	10	2	5

The correct pair will give $19x$ when multiplied across.

Hence: $6x^2 + 19x + 10 = (3x + 2)(2x + 5)$

Example 9

Factorise $4x^2 - 4x - 15$.

Solution

The factors of $4x^2$ are either $4x$ and x, or $2x$ and $2x$.

The factors of -15 are -15 and 1, 15 and -1, -5 and 3, or 5 and -3.

Set up the cross method:

$$\begin{array}{c c rrrr} 4x & \times & -15 & 15 & -5 & 5 \\ x & & 1 & -1 & 3 & -3 \end{array} \quad \text{or} \quad \begin{array}{c c rrrr} 4x & \times & 1 & -1 & 3 & -3 \\ x & & -15 & 15 & -5 & 5 \end{array}$$

or

$$\begin{array}{c c rrrr} 2x & \times & -15 & 15 & -5 & 5 \\ 2x & & 1 & -1 & 3 & -3 \end{array} \quad \text{or} \quad \begin{array}{c c rrrr} 2x & \times & 1 & -1 & 3 & -3 \\ 2x & & -15 & 15 & -5 & 5 \end{array}$$

The only combination that gives $-4x$ is:

$$\begin{array}{c c r} 2x & \times & -5 \\ 2x & & 3 \end{array}$$

Hence: $4x^2 - 4x - 15 = (2x - 5)(2x + 3)$

Example 10

Factorise $3x^2 + 8x - 16$.

Solution

The factors of $3x^2$ are $3x$ and x.

The factors of -16 are -16 and 1; 16 and -1; -8 and 2; 8 and -2; -4 and 4; or 4 and -4.

Set up the cross method:

$$\begin{array}{c c rrrrrrrrrrrr} 3x & \times & -16 & 1 & 16 & -1 & -8 & 2 & -2 & 8 & -4 & 4 \\ x & & 1 & -16 & -1 & 16 & 2 & -8 & 8 & -2 & 4 & -4 \end{array}$$

The only combination that gives $8x$ is:

$$\begin{array}{c c r} 3x & \times & -4 \\ x & & 4 \end{array}$$

Hence: $3x^2 + 8x - 16 = (3x - 4)(x + 4)$