RATES OF CHANGE WITH RESPECT TO TIME

1 When concentrated chemical solutions are allowed to evaporate slowly, crystals are formed. The surface area of a particular crystal is given by $A = 0.8t^2$, where A is mm^2 and t is days of evaporation. The rate at which the surface area is increasing after 4 days is:

A 0.8 mm^2 day^{-1} **B** 1.6 mm^2 day^{-1} $\boxed{\textbf{C}}$ 6.4 mm^2 day^{-1} **D** 12.8 mm^2 day^{-1}

$A(t) = 0.8\,t^2$

$$\frac{dA}{dt} = 0.8 \times 2t = 1.6\,t$$

so at $t = 4$ $\quad \frac{dA}{dt} = 1.6 \times 4 = 6.4$ mm^2/day

Response $\boxed{C}$

2 The length of the sides of a square, x cm, is given by $x = 4t + 1$ where t is in seconds.

(a) At what rate is the length of the side of the square increasing at t seconds?
(b) At what rate is the length of the side of the square increasing when $t = 5$ seconds?
(c) Write an expression for the area A cm^2 of the square as a function of t.
(d) At what rate is the area of the square increasing when $t = 5$ seconds?

a) $\frac{dx}{dt} = \frac{d}{dt}(4t+1) = 4$ cm s^{-1}

b) same, 4 cm s^{-1}, as $\frac{dx}{dt}$ does not depend of t.

c) $A = x^2 = (4t+1)^2 = 16t^2 + 8t + 1$

d) $\frac{dA}{dt} = 16 \times 2t + 8 = 32t + 8$

so at $t = 5$, $\frac{dA}{dt} = 32 \times 5 + 8 = 168$ cm^2/s.

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3 A petrol pump delivers petrol at the rate of 4 litres per minute.

(a) How much petrol is delivered in 5 minutes?

(b) If V litres of petrol are delivered in t minutes at this rate, find an expression for V in terms of t.

(c) How much time will it take to deliver 45 litres of petrol?

a) $\frac{dV}{dt} = 4 \text{ L min}^{-1}$

so at $t = 5$ minutes, $4 \times 5 = 20$ L have been delivered.

b) $V(t) = 4t$

c) For $V = 45$, then $45 = 4t$

so $t = \frac{45}{4} = 11.25$ minutes

or 11 minutes and 15 s.

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4 A tank holds 50 000 litres of water. The water drains from the tank in 40 minutes. The volume of water remaining in the tank after t minutes is given by $V = 50000\left(1 - \frac{t}{40}\right)^2$, where V is measured in litres.

(a) Find the rate at which the water is draining from the tank after: **(i)** 5 minutes **(ii)** 10 minutes **(iii)** 20 minutes.

(b) How much water remains in the tank after 20 minutes?

(c) How much time will it take until only half the initial volume of water remains in the tank?

a) $V(t) = 50000\left[1 - \frac{t}{40}\right]^2 = 50{,}000\left[1 - \frac{t}{20} + \frac{t^2}{1{,}600}\right]$

$V(t) = 50{,}000 - 2{,}500t + \frac{125}{4}t^2$

so $\frac{dV}{dt} = -2500 + \frac{125}{4} \times 2t = \frac{125}{2}t - 2{,}500$

i) At $t = 5$ $\quad \frac{dV}{dt} = \frac{125}{2} \times 5 - 2{,}500 = -2{,}187.5$ L/min

ii) At $t = 10$ $\quad \frac{dV}{dt} = \frac{125}{2} \times 10 - 2{,}500 = -1{,}875$ L/min

iii) $t = 20$ $\quad \frac{dV}{dt} = -1250$ L/min

b) At $t = 20$ $\quad V(20) = 50{,}000\left[1 - \frac{20}{40}\right]^2 = 50{,}000 \times \left[\frac{1}{2}\right]^2$

$V(20) = 12{,}500$ L

c) For V to be equal to $\frac{50{,}000}{2} = 25{,}000$,

$25{,}000 = 50{,}000\left[1 - \frac{t}{40}\right]^2 \iff \left[1 - \frac{t}{40}\right]^2 = \frac{1}{2}$

so $1 - \frac{t}{40} = \pm\frac{1}{\sqrt{2}}$ so $\frac{t}{40} = 1 \pm \frac{1}{\sqrt{2}}$

$t = 40\left[1 \pm \frac{1}{\sqrt{2}}\right]$ the first time is $t = 40\left[1 - \frac{1}{\sqrt{2}}\right]$

which is 11.71

so 11 minutes and 43 s.

5 The volume of water in a tank is given by $V = 1000 - 2t + \frac{t^2}{1000}$, where V is measured in litres and t is in minutes.

(a) How much water is in the tank initially, assuming it was full?

(b) Find an expression for $\frac{dV}{dt}$ as a function of t.

(c) At what rate is the water flowing out of the tank at 25 minutes?

(d) How much time will it take to empty the tank at this rate?

a) At $t=0$ $\quad V(0) = 1,000 - 2\times 0 + \frac{0^2}{1,000} = 1,000$ L initial volume.

b) $\frac{dV}{dt} = -2 + \frac{2t}{1,000} = \frac{t}{500} - 2$

c) at $t = 25$ $\quad \frac{dV}{dt} = \frac{25}{500} - 2 = -1.95$ L/min.

d) Assuming that the flow is -1.95 L / min, it would take $\frac{1000}{1.95} = 513$ minutes.

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6 The rate at which solvent in a nail polish evaporates is given by $\frac{dV}{dt} = \frac{1}{500}\left(1 - \frac{t}{60}\right)$, where V mL is the volume of solvent present and t is in seconds.

(a) What is the initial rate of evaporation of the solvent?

(b) When does the evaporation of the solvent stop?

a) At $t = 0$ $\quad \frac{dV}{dt} = \frac{1}{500}\left[1 - \frac{0}{60}\right] = \frac{1}{500}$ mL s^{-1}

b) $\frac{dV}{dt} = 0$ when $1 - \frac{t}{60} = 0$

i.e. $t = 60$ s.

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7 An electric current exists whenever electric charges move through a surface. The quantity of charge Q in coulombs (C) that has passed through a surface after time t, measured in seconds, is given by $Q = t^3 - 6t^2 + 12t + 5$. The current, in amperes (A), where $1\ A = 1\ C\,s^{-1}$ is given by $\frac{dQ}{dt}$. Find the current when:

(a) $t = 0.5$ s

(b) $t = 1$ s.

(c) What is the initial charge?

(d) When does the current stop flowing?

a) $\frac{dQ}{dt} = 3t^2 - 12t + 12$

At $t = 0.5$, $\frac{dQ}{dt} = 3 \times 0.5^2 - 12 \times 0.5 + 12$

$\frac{dQ}{dt} = 6.75$ A

b) At $t = 1$ $\frac{dQ}{dt} = 3 \times 1^2 - 12 \times 1 + 12 = 3$ A

c) At $t = 0$ $Q(0) = 0^3 - 6 \times 0^2 + 12 \times 0 + 5$

So $Q(0) = 5$ Coulomb.

d) The current stops flowing when $\frac{dQ}{dt} = 0$,

i.e. when $3t^2 - 12t + 12 = 0$

$\Leftrightarrow t^2 - 4t + 4 = 0$

$\Leftrightarrow (t-2)^2 = 0$

$\Rightarrow \boxed{t = 2 \text{ seconds.}}$