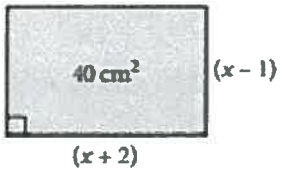
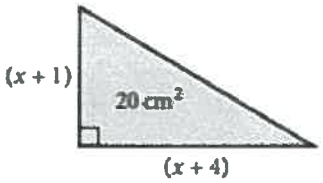
PROBLEMS INVOLVING QUADRATIC FUNCTIONS

1 In each diagram, all measurements are in centimetres and the area of the shaded region is given. Find the value of x in each case.

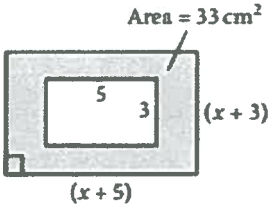
(a)



(b)



(c)



a) $(x+2)(x-1) = 40 \iff x^2 + x - 2 = 40$

$\iff x^2 + x - 42 = 0$

$\Delta = 1^2 + 4 \times 42 = 169 = 13^2$

$x = \dfrac{-1 \pm 13}{2}$ so $x = \dfrac{12}{2} = 6$ (the other solution is negative so not possible)

b) $\dfrac{1}{2}(x+1)(x+4) = 20 \iff (x+1)(x+4) = 40$

$\iff x^2 + 5x + 4 - 40 = 0$

$\iff x^2 + 5x - 36 = 0$

$\Delta = 25 - 4 \times (-36) = 169 = 13^2$

$x = \dfrac{-5 \pm 13}{2}$ so $x = 4$ (the other solution results in $x+1$ being negative, so not possible)

c) $(x+3)(x+5) - 15 = 33$

$x^2 + 8x + 15 - 15 = 33$

$x^2 + 8x - 33 = 0$ $\quad \Delta = 8^2 - 4 \times (-33) = 196 = 14^2$

So $x = \dfrac{-8 \pm 14}{2} = -4 \pm 7$

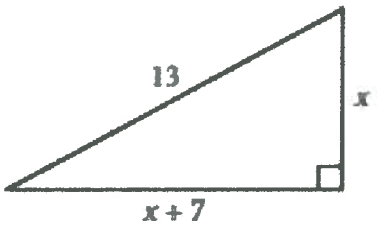
so $x = 3$

or $x = -11$ which is impossible, as it would result in $x + 3 < 0$

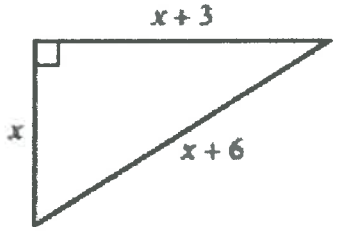
PROBLEMS INVOLVING QUADRATIC FUNCTIONS

3 Use Pythagoras' theorem to find the value of x, given that all measurements are in centimetres.

(a)



(b)



a) $13^2 = x^2 + (x+7)^2 = 2x^2 + 14x + 49$

$\Leftrightarrow \quad 2x^2 + 14x + 49 - 169 = 0$

$\Leftrightarrow \quad 2x^2 + 14x - 120 = 0$

$\Leftrightarrow \quad x^2 + 7x - 60 = 0$

$\Delta = 7^2 - 4 \times (-60) = 289 = 17^2$

$x = \dfrac{-7 \pm 17}{2} \quad$ so $\quad x = \dfrac{10}{2} = 5$ which is the only solution possible.

b) $(x+6)^2 = x^2 + (x+3)^2$

$\Leftrightarrow \quad \cancel{x^2} + 12x + 36 = \cancel{x^2} + x^2 + 6x + 9$

$\Leftrightarrow \quad x^2 - 6x - 27 = 0$

$\Delta = 6^2 - 4 \times (-27) = 144 = 12^2$

so $x = \dfrac{6 \pm 12}{2}$

so $x = \dfrac{12}{2} = 6$ (the other value is negative therefore impossible)

PROBLEMS INVOLVING QUADRATIC FUNCTIONS

5 The product of two numbers is 88. If one of the numbers is 3 more than the other, what are the numbers?

$x \times y = 88$ $\quad x = y + 3$

so $(y+3)y = 88 \iff y^2 + 3y - 88 = 0$

$\Delta = 3^2 - 4 \times (-88) = 361 = 19^2$

$y = \dfrac{-3 \pm 19}{2}$ so $y = -11$ or $y = 8$

$\downarrow$ $\quad$ $\downarrow$

so $\quad x = -8 \quad x = 11$

7 The height h metres of a stone, t seconds after being thrown straight up, is given by $h = 40t - 5t^2$. At what times is the stone at a height of: **(a)** 60 m **(b)** 80 m?

a) $60 = 40t - 5t^2 \iff 5t^2 - 40t + 60 = 0$

$\iff t^2 - 8t + 12 = 0$

$\Delta = 64 - 4 \times 12 = 16 = 4^2$

$t = \dfrac{8 \pm 4}{2} = 4 \pm 2$ so either $t = 2$ or $t = 6$

b) $80 = 40t - 5t^2 \iff 5t^2 - 40t + 80 = 0$

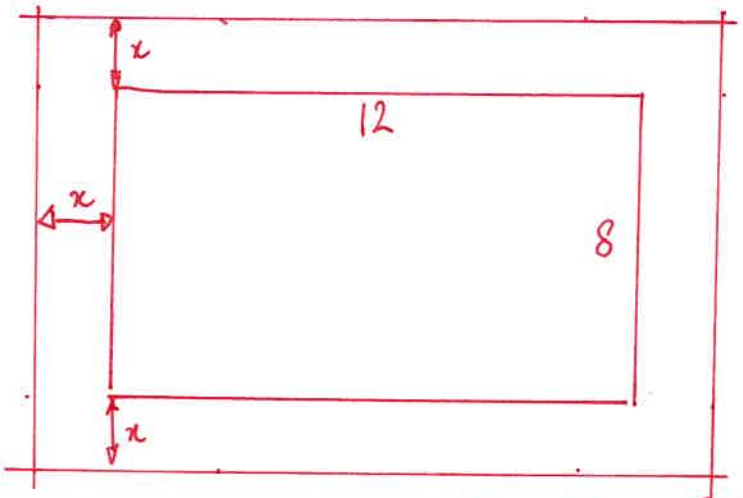
$\iff t^2 - 8t + 16 = 0$

$\Delta = 64 - 4 \times 16 = 0$ one solution

$t = \dfrac{8}{2} = 4$

PROBLEMS INVOLVING QUADRATIC FUNCTIONS

9 A rectangular swimming pool, 12 m by 8 m, is surrounded by a concrete path of uniform width. If the area of the path alone is 224 m^2, find its width.



Area of the path $= (8+2x) \times x \times 2 + 2 \times 12 \times x$

$= 2x(8+2x) + 24x$

$= 16x + 4x^2 + 24x$

So $4x^2 + 40x = 224$

$4x^2 + 40x - 224 = 0$

$x^2 + 10x - 56 = 0$

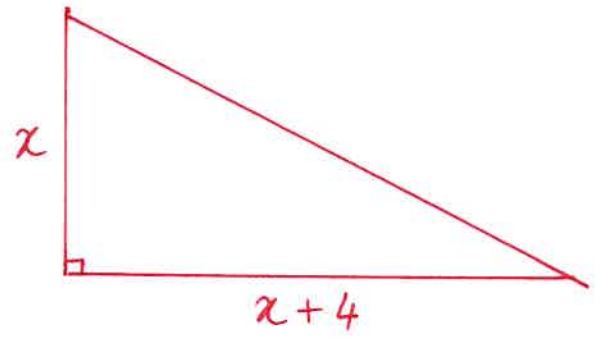
$\Delta = 10^2 - 4 \times (-56) = 324 = 18^2$

so $x = \dfrac{-10 \pm 18}{2}$

so $x = \dfrac{18 - 10}{2} = 4$ m

PROBLEMS INVOLVING QUADRATIC FUNCTIONS

13 In a right-angled triangle, one of the sides adjacent to the right angle is 4 cm longer than the other side. If the area of the triangle is 96 cm^2, find the length of each of the three sides.



$$\text{Area} = \frac{1}{2} \times x \times (x+4) = \frac{x^2+4x}{2}$$

$$\text{so } \frac{x^2+4x}{2} = 96 \iff x^2+4x = 192$$

$$\iff x^2+4x-192 = 0$$

$$\Delta = 4^2 - 4\times(-192) = 784 = 28^2$$

$$\text{so } x = \frac{-4 \pm 28}{2} = -2 \pm 14$$

so $x = 12$ cm , $x+4 = 16$ cm

and for the last side, we use Pythagoras.

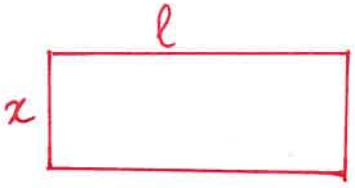
$$12^2 + 16^2 = 400$$

so the last side (hypotenuse) is $\sqrt{400} = 20$ cm

PROBLEMS INVOLVING QUADRATIC FUNCTIONS

14 The perimeter of a rectangle is 40 cm and its area is 84 cm^2.

(a) If the breadth of the rectangle is x cm, express the length in terms of x.
(b) Write the area of the rectangle in terms of x.
(c) Form a quadratic equation in x and solve it to find the length and breadth.



a) $2\ell + 2x = 40$ $\Rightarrow$ $\ell + x = 20$

$$\boxed{\ell = 20 - x}$$

b) $x \times (20 - x) =$ Area.

c) $x(20 - x) = 84$

$\Leftrightarrow -x^2 + 20x - 84 = 0$

$\Leftrightarrow x^2 - 20x + 84 = 0$

$\Delta = 20^2 - 4 \times 84 = 64 = 8^2$

$$x = \frac{20 \pm 8}{2} = 10 \pm 4$$

$\Rightarrow$ $x = 6$ m or $x = 14$ m

$\downarrow$ $\downarrow$

$\ell = 14$ m or $\ell = 6$ m

breadth is 6 m

length is 14 m