APPLICATIONS INVOLVING TRIGONOMETRIC FUNCTIONS AND GRAPHS

Trigonometric functions can be used to model periodic phenomena that occur in the physical world - that is, real-life situations that can be modelled by a function f where $f(x+b) = f(x)$ for all x and some non-real real number, b. The constant b is the period of the function. Average monthly temperatures, seasonal climate variations, sound waves, variation in tides and the phases of the moon are all real phenomena that display periodic behaviour and can be modelled by the sine or cosine functions.

In these situations, the independent variable on the graph is typically time rather than an angle. Because of this, the scale on the horizontal axis will usually not be represented in units that are exact fractions or multiples of π.

It is useful to sketch a graph of the trigonometric function and it may be necessary to determine the axes intercepts:

- to find the y-intercept, let $x = 0$ to find the corresponding y
- to find the x-intercept(s), let $y = 0$ and solve for x

Example 12

A Ferris wheel with a radius of 30 m rotates once every 60 seconds. Passengers get into a cabin 1 m above level ground, which is the wheel's lowest point. The height $h(t)$ metres above the ground is given by $h(t) = -30\cos\frac{\pi t}{30} + 31$, where t seconds is the time after leaving the lowest point.

(a) Sketch the graph of $h(t)$ for a 2-minute period after a passenger has got into the Ferris wheel.

(b) What is a passenger's height above the ground, to the nearest m, after 20 seconds?

(c) When will the cabin be 50 m above the ground for the first time?

Solution

(a) Write the value of k and its effect:

$k = -30$

The amplitude is $|-30| = 30$ metres.

Write the value of a and its effect:

$a = \frac{\pi}{30}$

The period is $2\pi \div \frac{\pi}{30} = 60$ seconds

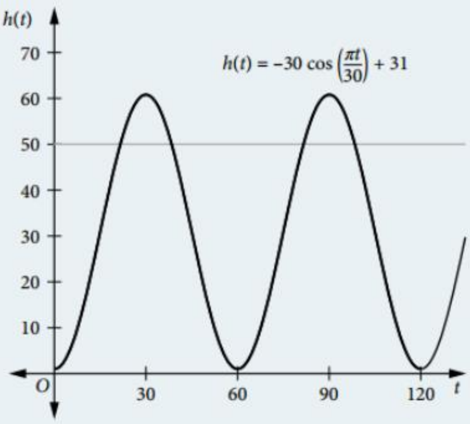
The graph will show 2 periods.

Write the value of c and its effect:

$c = 31$

The median value will be 31.

The minimum value of $h(t)$ will be $31 - 30 = 1$ metre and the maximum value of $h(t)$ will be $31 + 30 = 61$ metres.



(b) Substitute the given time into the equation to find the required height above the ground:

$$h(20) = -30\cos\frac{\pi \times 20}{30} + 31$$
$$\approx 46 \text{ metres}$$

(c) Solve the equation $50 = -30\cos\frac{\pi t}{30} + 31$ for t

$$\cos\frac{\pi t}{30} = -\frac{19}{30}$$
$$\frac{\pi t}{30} = \pi - \cos^{-1}\left(\frac{19}{30}\right)$$
$$\frac{\pi t}{30} = 2.2566$$
$$t = 21.55 \text{ seconds}$$

The cabin will be 50 m above the ground for the first time after approximately 21.55 seconds.

Example 13

The tide at a point on the NSW coast can be modelled using the equation $y = k\cos nt$. At Nobby's Beach in NSW, over two consecutive days, the average difference between high and low tides is 1.6 metres and the average time between high tide and low tide is 6.25 hours.

(a) What is the amplitude of the tide function at Nobby's Beach?

(b) How much time passes between successive high tides (i.e. the period) and what is the value of n?

(c) Use this information to obtain the Nobby's Beach tide function and draw its graph.

(d) If the depth of water at low tide is 0.2 metres, what is the depth of the water 2 hours after low tide?

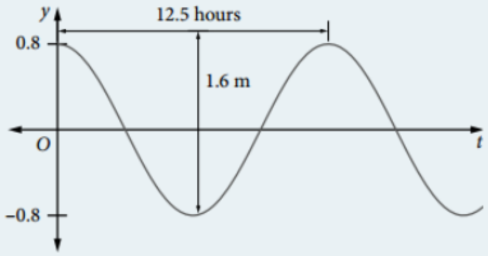
Solution

(a) $2a = 1.6$ so $a = 0.8$

(b) Time between successive high tides $= 2 \times 6.25 = 12.5$ hours

Period $T = \frac{2\pi}{n}$ so $12.5 = \frac{2\pi}{n}$ hence $n = \frac{4\pi}{25}$.

(c) $y = \frac{4}{5}\cos\frac{4\pi t}{25}$



(d) From the graph, low tide occurs at $t = 6.25$ and $y = -0.8$.

2 hours after low tide means $t = 6.25 + 2 = 8.25$

$t = 8.25$: $\quad y = \frac{4}{5}\cos\frac{33\pi}{25} = -0.43$ (to 2 d.p.)

Hence the depth is $0.8 - 0.43 = 0.37$ m above low tide.

$\therefore$ depth of the water 2 hours after low tide $= 0.2 + 0.37 = 0.57$ m (to 2 d.p.)