THE FIRST AND SECOND DERIVATIVE - CHAPTER REVIEW

1 For the graph of $y = 15x + 12x^2 - 4x^3$ for $-1 \le x \le 3$, find the values of x for which:

(a) y increases as x increases

(b) y decreases as x increases

(c) y is a maximum

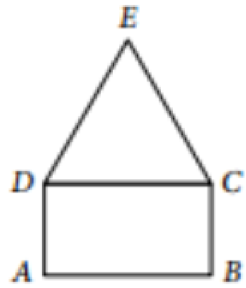
(d) y is a minimum.

3 Sketch the graph of $y = f(x)$, given that:

(a) $f(3) = 5$, $f'(3) = 0$, $f'(x) > 0$ for $x < 3$ and $f'(x) < 0$ for $x > 3$

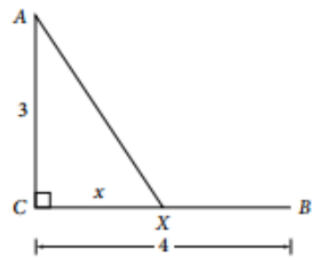
(b) $f(-1) = 8$, $f'(-1) = 0$, $f(2) = 3$, $f'(2) = 0$, $f'(x) < 0$ for $-1 < x < 2$, and $f'(x) > 0$ for $x < -1$ and for $x > 2$.

5 A figure *ABCED* consists of a rectangle *ABCD* topped by an equilateral triangle *CED* as shown in the diagram. If the perimeter of the figure is 45 cm, find the dimensions of the rectangle when the total area is a maximum.



10 A rectangular sheet of metal measures 6 cm by 4 cm. Four equal squares are cut out of the corners and the sides are turned up to form an open rectangular box. Find the edge length of the squares cut so that the box has a maximum volume.

13 Jack is in the bush at point A, 3 km from the nearest point C, which is at one end of a straight 4 km path CB, as shown in the diagram. Jack wants to get to point B, the other end of the path, as quickly as possible. He can run at a speed of $20\,\text{km}\,\text{h}^{-1}$ along the path CB but only at $10\sqrt{2}\,\text{km}\,\text{h}^{-1}$ in the bush off the path. He runs in a straight line through the bush from A to a point X on the path CB, then along the path from X to B.



(a) Find, in terms of x, the time taken for Jack to go from:

(i) A to X (ii) X to B.

(b) Find, in terms of x, the total time t hours to get from A to B.

(c) Find the position of the point X for which t is a minimum. Find this minimum time.

15 (a) Find the maximum value of $2xe^{-1.5x}$ and the value for which this function has a maximum value.
(b) If $f(x) = 2xe^{-1.5x}$, find $f(0)$, $f(0.5)$, $f(1)$ and hence graph the function in the domain $0 \le x \le 1$.

16 If $\theta = \theta_0 e^{-kt}$, show that $\frac{d\theta}{dt} = -k\theta$.