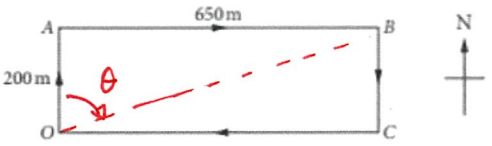
PROBLEMS INVOLVING DISPLACEMENT AND VELOCITY

1 Jaide is going for a walk around the block. She starts at O and moves to point A and then to point B, where she stops to have a rest.



(a) What is the distance Jaide has travelled from O to B?

(b) What is Jaide's displacement from O to B, correct to one decimal place, and what is her direction?

Jaide has taken 12 minutes to reach point B.

(c) What is Jaide's average speed in metres per second, correct to one decimal place?

(d) What is Jaide's average velocity in metres per second, correct to one decimal place, and what is her direction?

a) $d = 200 + 650 = 850$ m

b) $OB^2 = 200^2 + 650^2$ $\quad \therefore OB = \sqrt{200^2 + 650^2} = 680.1$ m

direction: $\tan\theta = \frac{650}{200}$ $\quad \therefore \theta = \tan^{-1}\left(\frac{650}{200}\right) = 72.9°$ E

c) Average speed $= \frac{850}{12 \times 60} = 1.2$ m s^{-1}

d) Average velocity $= \frac{680.1}{12 \times 60} = 0.9$ m s^{-1}

in the direction 72.9° E

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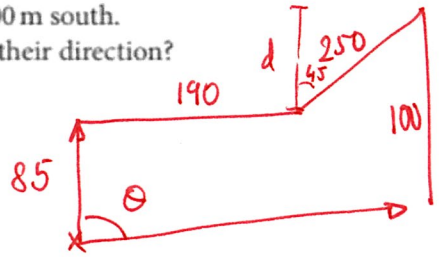
5 While exploring a recently discovered cave system, a spelunker (cave explorer) starts at the entrance and makes the following movements: 85 m north, 190 m east, 250 m N45°E, and 100 m south. What is the spelunker's final displacement from the cave entrance and what is their direction?

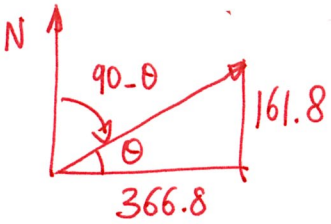


$d = 250 \cos 45 = \frac{\sqrt{2}}{2} \times 250 = 125\sqrt{2}$

So $\Delta y = 85 + 125\sqrt{2} - 100 = 161.8$ m N

$\Delta x = 190 + 125\sqrt{2} = 366.8$ m E

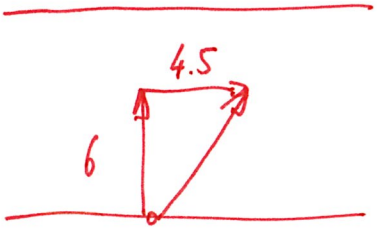
Total displacement $= \sqrt{(161.8)^2 + (366.8)^2} = 400.9$ m



Direction $\tan\theta = \frac{161.8}{366.8}$ so $\theta = 23.8°$

So direction is $(90 - 23.8°) = 66.2°$ E

6 Brianna can swim in still water at a rate of 6.0 km h^{-1}. If she swims in a river flowing at 4.5 km h^{-1} and keeps her direction (with respect to the water) perpendicular to the flow, then what is the magnitude of her velocity (correct to one decimal place) with respect to the riverbank?



$V = \sqrt{6^2 + 4.5^2} = 7.5$ km hr^{-1}

PROBLEMS INVOLVING DISPLACEMENT AND VELOCITY

9 (a) Raphaela is watching an aircraft that is flying at a velocity of $100\,m\,s^{-1}$ N with respect to the flow of air. If the velocity of the wind is $10\,m\,s^{-1}$ north, what is the resultant velocity of the aircraft relative to Raphaela?

(b) Later, she observes a second aircraft flying with a velocity of $125\,m\,s^{-1}$ N with respect to the flow of the air. If the flow of the air has a velocity of $10\,m\,s^{-1}$ south, then the resultant velocity of this aircraft is $115\,m\,s^{-1}$. What is the direction of this resultant velocity, relative to Raphaela?

(c) The next day, Raphaela observes another aircraft flying with a velocity of $100\,m\,s^{-1}$ N which encounters a wind coming from the side at a rate of $25\,m\,s^{-1}$ W. What is the resultant velocity of this aircraft relative to Raphaela, correct to one decimal place, and what is its direction?

a)

velocity of the aircraft relative to R $= 100 + 10$ (10 ↓)
$= 110\ m\,s^{-1}$

b) North.

c)

$\tan\theta = \frac{25}{100} = \frac{1}{4}$

$\theta = 14.0°$ W

$\sqrt{100^2 + 25^2} = 103.1\ m\,s^{-1}$

So the resultant velocity of this aircraft relative to Raphaela is $103.1\ m\,s^{-1}$ in the direction N14.0 W.

PROBLEMS INVOLVING DISPLACEMENT AND VELOCITY

11 Let $\underset{\sim}{i}$ and $\underset{\sim}{j}$ be unit vectors in the directions of east and north respectively. Give answers correct to two decimal places where required.

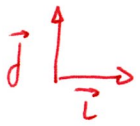
(a) Express each vector in the form $x\underset{\sim}{i} + y\underset{\sim}{j}$.

(i) $\overrightarrow{OA}$ = 5.0 m at 053°T (ii) $\overrightarrow{AB}$ = 6.0 m at 315°T (iii) $\overrightarrow{BC}$ = 4.0 m at 240°T (iv) $\overrightarrow{CD}$ = 3.0 m at 150°T.

(b) Express the sum of the four displacement vectors in the form $x\underset{\sim}{i} + y\underset{\sim}{j}$, with values correct to two decimal places.

(c) What is the magnitude and direction of the resultant vector $\overrightarrow{OD}$, correct to two decimal places, and what is its direction?

a) i)



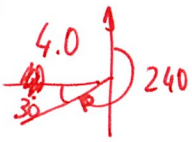
$x_{OA} = 5 \sin 53 \qquad y_{OA} = 5 \cos 53$

$\overrightarrow{OA} = 5 \sin 53\ \vec{i} + 5 \cos 53\ \vec{j} = 3.99\ \vec{i} + 3.01\ \vec{j}$

ii)

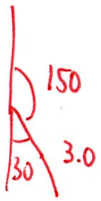
$\overrightarrow{AB} = -6 \sin 45\ \vec{i} + 6 \cos 45\ \vec{j} = -4.24\ \vec{i} + 4.24\ \vec{j}$

iii)



$\overrightarrow{BC} = -4 \cos 30\ \vec{i} - 4 \sin 30\ \vec{j} = -3.46\ \vec{i} - 2\ \vec{j}$

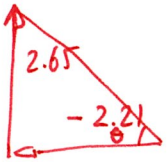
iv)



$\overrightarrow{CD} = 3 \sin 30\ \vec{i} - 3 \cos 30\ \vec{j} = 1.5\ \vec{i} - 2.60\ \vec{j}$

b) $\text{Sum} = -2.21\ \vec{i} + 2.65\ \vec{j} = \overrightarrow{OD}$

c) $|\overrightarrow{OD}| = \sqrt{2.21^2 + 2.65^2} = 3.45$

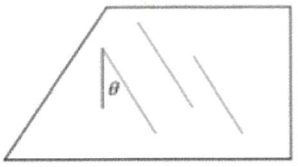


$\tan\theta = \dfrac{2.65}{2.21} \qquad \therefore \theta = \tan^{-1}\left(\dfrac{2.65}{2.21}\right) = 50.17°$

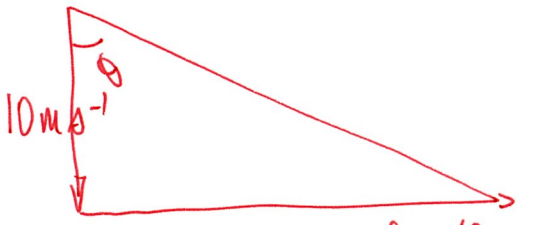
or direction is $(270 + 50.17°) = 320.17°$ T

PROBLEMS INVOLVING DISPLACEMENT AND VELOCITY

13 Johanna is driving along the freeway at $108\,\text{km h}^{-1}$ when it begins to rain. She observes the raindrops running down the driver's side window.



Calculate the angle that the raindrops make with the vertical window, as seen by Johanna, if the raindrops have a speed of $10\,\text{m s}^{-1}$ relative to the Earth's surface. Assume that the raindrops fall vertically down, relative to the Earth's surface.



or 30 m/s

$$\tan\theta = \frac{30}{10} = 3$$

$$\theta = \tan^{-1} 3 = 71.6^\circ$$

PROBLEMS INVOLVING DISPLACEMENT AND VELOCITY

17 The pilot of an aircraft flying due south is notified by the flight controller that there is a second aircraft flying due north in the same general area and at the same altitude. The pilot is told that the northbound aircraft is currently located at a position that is 12.6 km, 170°T with respect to the pilot's aircraft.

(a) How many kilometres to the south is the second aircraft?
(b) How many kilometres to the east is the second aircraft?
(c) If the two aircrafts both have an airspeed of 300 km h^{-1}, how much time (in seconds) will elapse before they are *side by side*?

a) $d_s = 12.6 \times \cos 10$

$d_s = 12.4$ km

The second aircraft is 12.4 km to the south

b) $d_E = 12.6 \times \sin 10$

$d_E = 2.2$ km

The second aircraft is 2.2 km to the east

c) relative airspeed = 600 km hr^{-1}

So $t = \frac{12.4}{600} = 0.0206\dot{}$ hr or 74.4 seconds.

74.4 s will have elapsed before the aircrafts are side by side.