FURTHER INTEGRATION - CHAPTER REVIEW

1 Evaluate: **(a)** $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} x \cos x \, dx$ **(b)** $\int_{3}^{4} \frac{5x-7}{x^2-3x+2} dx$

2 Find: **(a)** $\int \log_e 2x\,dx$ **(b)** $\int \frac{x+2}{x^2-1}\,dx$

4 Evaluate: **(a)** $\int_{1}^{\sqrt{3}} \tan^{-1} x \, dx$ **(b)** $\int_{-2}^{2} \frac{6}{9 - x^2} dx$

5 Find the derivative of $\log_e(\operatorname{cosec} x + \cot x)$ and deduce the value of:

(a) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{cosec}\frac{\theta}{2}\, d\theta$ **(b)** $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec u\, du$

6 **(a)** Find the derivative of: $\dfrac{\sin x}{1-\sin^2 x}+\log_e\sqrt{\dfrac{1+\sin x}{1-\sin x}}$

(b) Hence evaluate: $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^3\theta\, d\theta$

7 **(a)** Write $(x-1)(7-x)$ in the form $b^2-(x-a)^2$, where a and b are real numbers.

(b) Using the values of a and b from part **(a)** and making the substitution $x-a=b\sin\theta$, or otherwise, evaluate: $\int_1^7 \sqrt{(x-1)(7-x)}\,dx$

9 Reduce each rational function to its partial fractions.

(a) $\dfrac{x^2-10x+13}{(x-1)(x^2-5x+6)}$ (b) $\dfrac{x^2+10x+16}{(x-1)(x^2-4)}$

13 Evaluate:

(a) $\displaystyle\int_1^3 \frac{2x^2+2x+5}{(x^2+3)(2x-1)}\,dx$ **(b)** $\displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x\cos x\,dx$

14 Differentiate $\log_e x - \log_e\left(a+\sqrt{a^2-x^2}\right)$ where $a>0$ and deduce the value of: $\int_3^4 \frac{dx}{x\sqrt{25-x^2}}$

15 Evaluate: **(a)** $\int_1^4 (x+1)\sqrt{x}\,dx$ **(b)** $\int_0^{\frac{\pi}{2}} \cos x\, e^{\sin x}\,dx$ **(c)** $\int_5^6 \frac{dx}{x^2-16}$

15 Evaluate: **(d)** $\int_{-1}^{1}(2x-1)\sin x\,dx$ **(e)** $\int_{0}^{3}x^2\sqrt{9-x^2}\,dx$

16 Find: **(a)** $\int \frac{dx}{1-4x^2}$ **(b)** $\int \frac{x}{1-4x^2}\,dx$ **(c)** $\int \frac{x^2}{1-4x^2}\,dx$

16 Find: **(d)** $\int \frac{x}{\sqrt{1-4x^2}}\,dx$ **(e)** $\int \frac{dx}{\sqrt{1-4x^2}}$ **(f)** $\int \frac{dx}{1+4x^2}$

17 Find: (a) $\int \frac{dx}{\sin x + \tan x}$ (b) $\int \frac{dx}{5 + 4\cos 2x}$

17 Find: **(c)** $\int \frac{d\theta}{4\cos\theta - 3\sin\theta}$

18 Use the substitution $t = \tan\frac{x}{2}$ to find the exact value of $\int_0^{\frac{\pi}{3}} \frac{1}{4+5\cos x}\,dx$.

19 Find: $\int x \log_e 2x \, dx$

20 Find: (a) $\int \frac{x^2+1}{x^3+3x}dx$ (b) $\int \frac{dx}{x^2+2x+1}$

20 Find: **(c)** $\int \frac{x^3+1}{x}\,dx$ **(d)** $\int \frac{x+1}{\sqrt{x^2+2x-3}}\,dx$

20 Find: (e) $\int \frac{x+4}{x^3+4x}\,dx$ (f) $\int \frac{dx}{x^3-1}$

20 Find: (g) $\int x \sin^{-1} x \, dx$

21 Find: **(a)** $\int \frac{dx}{x^2 - 4x - 1}$ **(b)** $\int \frac{dx}{3x^2 + 6x + 10}$

21 Find: (c) $\int \frac{dx}{\sqrt{x^2 - 4x + 1}}$ (d) $\int \frac{dx}{\sqrt{x^2 + 16}}$

23 Calculate the area of the region bounded by the curve $y = xe^{-x}$, the x-axis and the line $x = 1$.

24 Sketch the graph of $y = \dfrac{\cos x}{1 + \cos x}$ for $-\pi < x < \pi$, stating the coordinates of its intersection with the x-axis and of the turning point. Find the area of the region bounded by the curve and the x-axis.

27 **(a)** Using the substitution $u = a - x$, or otherwise, prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

(b) Hence evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\,dx$.