OTHER REPRESENTATIONS OF COMPLEX NUMBERS

1 Write each complex number in both polar and Cartesian form.

(a) $e^{\frac{i\pi}{3}}$ **(b)** $e^{\frac{i\pi}{2}}$ **(c)** $e^{\frac{5\pi i}{6}}$ **(d)** $e^{\frac{i\pi}{4}}$ **(e)** $e^{\frac{-i\pi}{2}}$ **(f)** $e^{\frac{-2\pi i}{3}}$ **(g)** $e^{1-\frac{i\pi}{2}}$ **(h)** $e^{2+\frac{i\pi}{3}}$

OTHER REPRESENTATIONS OF COMPLEX NUMBERS

2 Write each complex number in the form $re^{i\theta}$, giving any decimal answers correct to two decimal places, where necessary.

(a) $3(\cos 1.5 + i\sin 1.5)$ **(b)** $-\sqrt{3} + i$ **(c)** $3 + 2i$ **(d)** $4(\cos 2 - i\sin 2)$

(e) $2 - 2i$ **(f)** $4\left(-\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$ **(g)** $-2 - 2\sqrt{3}i$ **(h)** $\left(1+\sqrt{2}\right)+\left(1-\sqrt{2}\right)i$

OTHER REPRESENTATIONS OF COMPLEX NUMBERS

3 If $\cos\frac{\pi}{4} = 2\cos^2\frac{\pi}{8} - 1$, then the complex number $\frac{1}{2}\left(\sqrt{2+\sqrt{2}} + i\sqrt{2-\sqrt{2}}\right)$ is equal to:

A $e^{\frac{5\pi i}{8}}$ **B** $e^{\frac{-5\pi i}{8}}$ **C** $e^{\frac{i\pi}{8}}$ **D** $e^{\frac{-i\pi}{8}}$

4 **(a)** Given that $e^{i\theta} = \cos\theta + i\sin\theta$, write an expression for $e^{-i\theta}$.

(b) Using part **(a)**, obtain expressions for $\sin\theta$ and $\cos\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$.

OTHER REPRESENTATIONS OF COMPLEX NUMBERS

6 **(a)** Write $z = 1 - \sqrt{3}i$ in the form $re^{i\theta}$.

(b) Hence find the following in both polar form and Cartesian form.

(i) z^2 **(ii)** z^3 **(iii)** z^5 **(iv)** $\sqrt{z}$ **(v)** $\frac{1}{z}$

9 **(a)** Given $e^{i\theta} = \cos\theta + i\sin\theta$, write expressions for $e^{3i\theta}$ and $\left(e^{i\theta}\right)^3$.

(b) Hence write expressions for $\cos 3\theta$ in terms of $\cos\theta$ and $\cos^3\theta$.

(c) Hence write expressions for $\sin 3\theta$ in terms $\sin\theta$ and $\sin^3\theta$.

10 Given that $\dfrac{1-3i}{1+2i} = re^{i\theta}$, $r > 0$ and for $-\pi < \theta \le \pi$, find the values of r and θ.

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14 Given $z_1 = 2e^{\frac{i\pi}{8}}$, $z_2 = 3e^{\frac{5i\pi}{12}}$, $z_3 = \frac{1}{3}e^{\frac{-5i\pi}{6}}$ and $z_4 = \frac{1}{2}e^{\frac{-3i\pi}{4}}$, find the polar form for each of the following, plotting each one on the Argand plane.

(a) $z_1^{\,2} \times z_4$ **(b)** $z_2 \times z_3$ **(c)** $z_1 \times z_2 \times z_3 \times z_4$ **(d)** $\frac{z_1^{\,2}}{z_4}$ **(e)** $\frac{\sqrt{z_3}}{z_2}$

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16 $OABC$ is a square on an Argand diagram. O represents 0, A represents $-4 + 2i$, B represents z, C represents w and D is the point where the diagonals of the square meet. Note that there are two squares that satisfy these requirements. For each square, find:

(a) the complex numbers represented by C and D in Cartesian form

(b) the value of $\arg\left(\frac{w}{z}\right)$.

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17 On an Argand diagram, $OABC$ is a rectangle. The length of OC is twice the length of OA. The vertex A corresponds to the complex number z. Find the complex number represented by D, the point of intersection of the diagonals OB and AC.