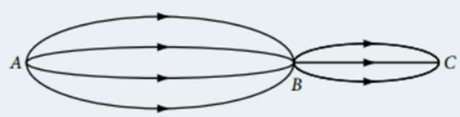
# FUNDAMENTAL COUNTING PRINCIPLE

**Problem 1:** If you have 5 different shirts and 7 different hats, then the total number of different selections of a shirt and hat is $5 \times 7 = 35$.

This is known as the **multiplication principle of combinatorics**: if an outcome can happen in $m$ different ways, and a second outcome can happen in $n$ different ways, then the total number of ways in which the two outcomes can happen together is $mn$.

**Problem 2:**

There are four roads from town $A$ to town $B$, and three roads from town $B$ to town $C$.
How many different ways are there to travel by road from $A$ to $B$ to $C$?




Solution

Consider the first road from $A$ to $B$. After this road there are three ways to travel from $B$ to $C$.

Similarly, after taking the second road from $A$ to $B$ there are then three ways to go from $B$ to $C$.

As there are 4 different ways from $A$ to $B$, and after each of these there are 3 different ways to go from $B$ to $C$, there are in total $4 \times 3 = 12$ ways to go from $A$ to $B$ to $C$ (i.e. from $A$ to $C$ via $B$).

**Problem 3:** The old New South Wales black-and-yellow number plate system showed three letters and three digits. How many different number plates could be made?

There are 26 letters of the alphabet and 10 digits to choose from, so the number of different number plates is $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 26^3 \times 10^3 = 17{,}576{,}000$

**Problem 4.a):** How many ways can six people A, B, C, D, E, F be arranged in a row of 3 seats?

- There is a choice of 6 people for the 1st place.
- There is a choice of 5 people for the 2nd place, as one person already in the first seat.
- There is a choice of 4 people for the 3rd place, as two people have already been chosen.

Using the fundamental counting principle, this gives $6 \times 5 \times 4 = 120$ possible different ways that six people can be arranged in a row of 3 seats.

# FUNDAMENTAL COUNTING PRINCIPLE

**Problem 4.b):** How many ways can six people A, B, C, D, E, F be arranged in a row of 6 seats?

- There is a choice of 6 people for the 1st place.
- There is a choice of 5 people for the 2nd place, as one person already in the first seat.
- There is a choice of 4 people for the 3rd place, as two people have already been chosen.
- There is a choice of 3 people for the 4th place, as three people have already been chosen.
- There is a choice of 2 people for the 5th place, as four people have already been chosen.
- There is a choice of 1 people for the 6th place.

Using the fundamental counting principle, this gives $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ possible different ways that six people can be arranged in a row of 6 seats.

**Factorial notation:** $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$

Using the factorial notation, the answer to Problem 4.b) above can be rewritten as 6!

An alternative definition of $n!$ is the **recursive** definition: $n! = n \times (n-1)!$ (with $n$ being a natural number greater than 0). Particularly, for $n = 1$, we obtain:

$1! = 1 \times 0!$ therefore $0! = \frac{1!}{1} = \frac{1}{1} = 1$ and so $\mathbf{0! = 1}$

### Example 4

A group of 6 boys and 5 girls decide to go to the movies.

**(a)** In how many ways can the boys sit together in a row?

**(b)** In how many ways can the girls sit together in a row?

**(c)** In how many ways can the whole group sit together in a row?

**(d)** Why is the answer to part **(c)** not the product of the answers in parts **(a)** and **(b)**?

### Solution

**(a)** The 6 boys can sit together in 6! ways, i.e. 720 ways.

**(b)** The 5 girls can sit together in 5! ways, i.e. 120 ways.

**(c)** The group of 11 people can sit together in 11! ways, i.e. 39 916 800 ways.
(*Note:* This answer is best written simply as '11!' rather than as '39 916 800'.)

**(d)** In part **(c)** there is no grouping by gender, so 11 different people are being arranged in one big group. But multiplying the answers to parts **(a)** and **(b)** would keep the two groups separate, only finding half the number of ways a group of 6 people can sit in a row with a group of 5 people.