FINITE GEOMETRIC SERIES

1 Find the sum of the first six terms of the series 4 + 6 + 9 + …

$\frac{6}{4} = \frac{3}{2} = \frac{9}{6}$ ∴ This is therefore a geometric series of first term 4 and common ratio 3/2.

$$S_n = \frac{a(r^n - 1)}{r-1} \text{ as } r>1$$

$$S_6 = \frac{4\left[\left(\frac{3}{2}\right)^6 - 1\right]}{\frac{3}{2} - 1} = \frac{665}{8} = 83\frac{1}{8}$$

2 Find the sum of the first ten terms of the series 8 − 4 + 2 − …

$\frac{-4}{8} = -\frac{1}{2}$ whereas $\frac{2}{-4} = -\frac{1}{2}$ ∴ This is a geometric series of first term 8 and common ratio $(-1/2)$, which is less than 1

$$S_{10} = \frac{8\left(1-\left(-\frac{1}{2}\right)^{10}\right)}{1-\left(-\frac{1}{2}\right)} = \frac{341}{64} = 5\frac{21}{64}$$

3 The sum of the first ten terms of the series $2 + 2\sqrt{3} + 6 + \ldots$ is:

$\boxed{\text{A}}$ $242(\sqrt{3}+1)$ B $162\sqrt{3}$ C $244(\sqrt{3}+1)$ D $121(\sqrt{3}+1)$

$\frac{2\sqrt{3}}{2} = \sqrt{3}$ whereas $\frac{6}{2\sqrt{3}} = \sqrt{3}$ so it's a geometric series of common ratio $r = \sqrt{3}$ and 1st term is 2

$$S_{10} = \frac{2\left[(\sqrt{3})^{10} - 1\right]}{\sqrt{3}-1} = \frac{2[243-1]}{\sqrt{3}-1}$$

$$S_{10} = \frac{484[\sqrt{3}+1]}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{484[\sqrt{3}+1]}{2} = 242[\sqrt{3}+1]$$

$\boxed{A}$

FINITE GEOMETRIC SERIES

4 Evaluate the series $16 - 8 + 4 - 2 + \ldots + \frac{1}{16}$.

$\frac{-8}{16} = -\frac{1}{2}$ whereas $\frac{4}{-8} = -\frac{1}{2}$ and $\frac{-2}{4} = -\frac{1}{2}$

So it's a geometric series of common ratio $(-\frac{1}{2})$ and 1st term 16

$T_n = a r^{n-1} = 16 \times \left(\frac{-1}{2}\right)^{n-1}$

$T_n = \frac{1}{16}$ when $\frac{1}{16} = 16 \times \left(-\frac{1}{2}\right)^{n-1}$ i.e. when $\left(\frac{-1}{2}\right)^{n-1} = \frac{1}{2^8}$

so when $n-1 = 8$ or $n = 9$ So $\frac{1}{16}$ is the 9th term of the series

$$S_9 = \frac{16\left[1-(-\frac{1}{2})^9\right]}{\left[1-(-\frac{1}{2})\right]} = \frac{171}{16} = 10\frac{11}{16}$$

7 How many terms of the series $6 + 3 + \frac{3}{2} + \ldots$ must be taken to give a sum of $11\frac{13}{16}$? Indicate whether each statement below is a correct or incorrect step in solving this problem.

$\boxed{\text{(a)}}$ $a = 6, r = \frac{1}{2}$ (b) $\frac{189}{16} = 12\left(1 - \frac{1}{2^n}\right)$ (c) $\frac{1}{2^n} = 1 + \frac{63}{64}$ (d) $n = 6$

$\frac{3}{6} = \frac{1}{2}$ whereas $\frac{3/2}{3} = \frac{1}{2}$ so $r = \frac{1}{2}$ and $a = 6$. $\boxed{a}$ is true.

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{6\left[1-(\frac{1}{2})^n\right]}{1-\frac{1}{2}} = 12\left[1-\frac{1}{2^n}\right]$$

$11\frac{13}{16} = \frac{189}{16}$ so indeed $\frac{189}{16} = 12\left[1 - \frac{1}{2^n}\right]$ $\boxed{b}$ is true.

$\Longleftrightarrow \frac{12}{2^n} = 12 - \frac{189}{16} = \frac{3}{16}$ $\therefore$ $\frac{1}{2^n} = \frac{3}{16 \times 12} = \frac{1}{64}$ $\boxed{c}$ not true

From $\frac{1}{2^n} = \frac{1}{64}$ we obtain $n = 6$ so $\boxed{d}$ is true.

8 Find the sum of the first ten terms of the series $\log_{10}3 + \log_{10}6 + \log_{10}12 + \ldots$

$\log_{10}6 - \log_{10}3 = \log_{10}\frac{6}{3} = \log_{10}2$

whereas $\log_{10}12 - \log_{10}6 = \log_{10}\frac{12}{6} = \log_{10}2$

So this is an arithmetic series of common difference $\log_{10}2$

For an arithmetic series $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{10} = \frac{10}{2}\left[2\log_{10}3 + (10-1) \times \log_{10}2\right]$

$S_{10} = 10\log_{10}3 + 45\log_{10}2$

9 The sum of the first eight terms of a geometric series is seventeen times the sum of its first four terms. Find the common ratio.

$S_8 = 17 \times S_4 \quad \therefore \quad \frac{a(1-r^8)}{1-r} = 17a\frac{(1-r^4)}{1-r}$ if $r<1$

$\therefore (1-r^8) = 17(1-r^4)$

$\therefore (1-r^4)(1+r^4) = 17(1-r^4) \quad \therefore 1+r^4 = 17$

$\therefore r^4 = 17-1 = 16 \quad \therefore \boxed{r=-2}$ ($r=+2$ is impossible as $r<1$)

OR $\frac{a(r^8-1)}{r-1} = 17\frac{a(r^4-1)}{r-1}$ if $r>1$

$\therefore (r^8-1) = 17(r^4-1) \Leftrightarrow r^4+1 = 17$

So $r^4 = 16$ so $\boxed{r=2}$ ($r=-2$ is impossible as we have assumed that $r>1$)

So two solutions $r = \pm 2$

10 An A0-sized sheet of paper is folded along its long side and then cut to create two sheets of A1-sized paper. Each sheet of A1-sized paper is folded along its long side and then cut to create two sheets of A2-sized paper. This process is repeated many times.

(a) How many sheets of A3-sized paper are created?

(b) You have eleven sheets of A0-sized paper. You cut each sheet into one of the eleven different sizes A0, A1, A2, … A10, creating as many sheets of each size as possible from each sheet of A0.

(i) How many sheets of A10 are created?

(ii) You stack all the sheets of paper on top of each other. with the A0 sheet on the bottom and the A10 sheets on top. How many sheets of paper are in the pile?

(iii) If a pack of 500 sheets of A4 paper is 55 mm thick, approximately how high is the stack of sheets in part (ii)?

a) let N_n the number of sheets of size A_n

$N_0 = 1$ and $N_n = 2N_{n-1}$ (as $N_1 = 2N_0$)

$N_3 = 2N_2 = 2 \times 2 N_1 = 2 \times 2 \times 2 N_0 = 8$.

b) i) $S = N_0 + N_1 + N_2 + \dots + N_{10}$

$N_{10} = 2^{10} \times N_0 = 1024$

ii) $S = N_0 + N_1 + N_2 + \dots + N_{10}$ ← 11th term

$$S = \frac{1 \times (2^{10+1} - 1)}{2-1} = 2047$$

iii) The pack would be $\frac{2047}{500} \times 55 = 225$ mm thick

that's 22.5 cm approx.