DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

1 Differentiate with respect to x:

(a) $\sin 3x$ **(b)** $3\sin x$ **(c)** $\cos 2x$ **(d)** $2\cos x$

a) $f'(x) = \cos 3x \times 3 = 3\cos 3x$ b) $f'(x) = 3\cos x$

c) $f'(x) = 2 \times (-\sin 2x) = -2\sin 2x$

d) $f'(x) = 2 \times (-\sin x) = -2\sin x$

(e) $\sin x + 4\cos x$ **(f)** $\tan 2x$ **(g)** $\sin 2x - \cos 2x$ **(h)** $\sin\left(x + \frac{\pi}{4}\right)$

e) $f'(x) = \cos x - 4\sin x$ f) $f(x) = \sec^2 2x \times 2 = 2\sec^2 2x$

g) $f'(x) = 2\cos 2x - 2 \times (-\sin 2x) = 2(\cos 2x + \sin 2x)$

h) $f'(x) = \cos\left(x + \frac{\pi}{4}\right) \times 1 = \cos\left(x + \frac{\pi}{4}\right)$

2 The derivative of $\cos^2 5t$ is:

$\boxed{\textbf{A}}$ $-10\sin 5t\cos 5t$ **B** $-10\cos 5t$ **C** $-5\sin 5t\cos 5t$ **D** $-2\sin 5t\cos 5t$

$f'(t) = 2\cos 5t \times (\cos 5t)' = 2\cos 5t \times (-5\sin 5t) = -10\sin 5t\cos 5t$

3 Differentiate with respect to x:

(a) $\sin x\cos x$ **(b)** $x\sin x$ **(c)** $2x\tan x$ **(d)** $x^2\cos x$

a) $u(x) = \sin x$ $u'(x) = \cos x$
$v(x) = \cos x$ $v'(x) = -\sin x$
$f'(x) = \cos^2 x - \sin^2 x = \cos 2x$

b) $u(x) = x$ $u'(x) = 1$
$v(x) = \sin x$ $v'(x) = \cos x$
$f'(x) = \sin x + x\cos x$

c) $u(x) = 2x$ $u'(x) = 2$
$v(x) = \tan x$ $v'(x) = \sec^2 x$
$f'(x) = 2\tan x + 2x\sec^2 x$

d) $u(x) = x^2$ $u'(x) = 2x$
$v(x) = \cos x$ $v'(x) = -\sin x$

$f'(x) = -x^2\sin x + 2x\cos x$

$f'(x) = x\left[-x\sin x + 2\cos x\right]$

DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

(i) $x\sec x$ (j) $\dfrac{\operatorname{cosec} x}{x}$ (k) $x^2\cot x$ (l) $\dfrac{\sec x}{\operatorname{cosec} x}$

i) $f(x) = \dfrac{x}{\cos x}$ $u(x) = x$ $u'(x) = 1$

$v(x) = \cos x$ $v'(x) = -\sin x$

$$f'(x) = \frac{1 \times \cos x - (-\sin x)x}{\cos^2 x} = \frac{\cos x + x\sin x}{\cos^2 x} = \frac{1}{\cos x} + \frac{x}{\cos x} \times \tan x$$

$$f'(x) = \sec x + x \sec x \tan x$$

j) $f(x) = \dfrac{1}{x\sin x} = (x\sin x)^{-1}$

if $g(x) = x \sin x = u(x) \times v(x)$

$u(x) = x$ $u'(x) = 1$ $(x\sin x)' = \sin x + x\cos x$

$v(x) = \sin x$ $v'(x) = \cos x$

$$f'(x) = (-1) \times (x\sin x)^{-2} \times (\sin x + x\cos x)$$

$$f'(x) = \frac{-\sin x - x\cos x}{x^2\sin^2 x} = \frac{-1}{x^2\sin x} - \frac{\cos x}{x^2\sin^2 x}$$

$$f'(x) = \frac{-\sec x}{x^2} - \frac{1}{x^2}\operatorname{cotan} x \csc x$$

k) $u(x) = x^2$ $u'(x) = 2x$

$v(x) = \operatorname{cotan} x = \dfrac{\cos x}{\sin x}$ $v'(x) = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\csc^2 x$

$$f'(x) = 2x\operatorname{cotan} x + x^2(-\csc^2 x) = 2x\operatorname{cotan} x - x^2\csc^2 x$$

l) $\dfrac{\sec x}{\operatorname{cosec} x} = \dfrac{1}{\cos x} \times \sin x = \tan x$

so $f'(x) = \sec^2 x$

DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

5 Differentiate with respect to x:

(a) $\cos^2 2x$ (b) $\sin^2 3x$ (c) $\cos^3 x$ (d) $\cos(x^3)$

a) $f'(x) = 2\cos 2x \times (\cos 2x)' = 2\cos 2x \times (-\sin 2x \times 2)$

$f'(x) = -4\sin 2x \cos 2x = -2\sin 4x$

b) $f'(x) = 2\sin 3x\,(\sin 3x)' = 2\sin 3x \times \cos 3x \times 3 = 3\sin 6x$

c) $f'(x) = 3\cos^2 x \times (\cos x)' = 3\cos^2 x \times (-\sin x)$

d) $f'(x) = -\sin(x^3) \times 3x^2$

$f'(x) = -3x^2 \sin(x^3)$

6 Find $f'(x)$ for $f(x) = 3\sin\frac{x}{2} - 4\cos\frac{3x}{2} - x^3$. Indicate whether each statement below is a correct or incorrect step in finding $f'(x)$.

(a) $\frac{d}{dx}\left(3\sin\frac{x}{2}\right) = \frac{3}{2}\cos\frac{x}{2}$ yes

(b) $\frac{d}{dx}\left(4\cos\frac{3x}{2}\right) = 6\sin\frac{3x}{2}$ NO

(c) $f'(x) = \frac{3}{2}\cos\frac{3x}{2} - 6\sin\frac{3x}{2} - 3x^2$ NO

(d) $f'(x) = \frac{3}{2}\cos\frac{x}{2} + 6\sin\frac{3x}{2} - 3x^2$ yes

$$f'(x) = 3 \times \frac{1}{2} \cos\left(\frac{x}{2}\right) - 4 \times \frac{3}{2}\left(-\sin\frac{3x}{2}\right) - 3x^2$$

7 Differentiate with respect to x:

(a) $\sqrt{\sin 2x}$ (b) $(\sin x - \cos x)^2$ (c) $\sin^2 x + \cos^2 x$

a) $f(x) = (\sin 2x)^{1/2}$ $f'(x) = \frac{1}{2}(\sin 2x)^{-1/2} \times (\sin 2x)'$

$$f'(x) = \frac{1}{2\sqrt{\sin 2x}} \times 2\cos 2x = \frac{\cos 2x}{\sqrt{\sin 2x}}$$

b) $(\sin x - \cos x)^2 = \sin^2 x - 2\sin x\cos x + \cos^2 x$

$= 1 - 2\sin x \cos x = 1 - \sin 2x$

So $f'(x) = -2\cos 2x$

c) $\sin^2 x + \cos^2 x = 1$

so $f'(x) = 0$

8 Differentiate with respect to x:

(a) $e^x \sin x$ **(b)** $e^{2x} \cos \frac{x}{2}$ **(c)** $e^{-x} \sin 3x$ **(d)** $e^x \cos 4x$

a) $f'(x) = e^x \cos x + e^x \sin x = e^x(\sin x + \cos x)$

b) $f(x) = u(x) \times v(x)$ $\quad u(x) = e^{2x}$ $\quad u'(x) = 2e^{2x}$

$v(x) = \cos\left(\frac{x}{2}\right)$ $\quad v'(x) = \frac{1}{2}\left(-\sin\frac{x}{2}\right)$

$$f'(x) = \frac{-e^{2x} \sin x/2}{2} + 2e^{2x} \cos\left(\frac{x}{2}\right)$$

$$f'(x) = e^{2x}\left[2\cos\frac{x}{2} - \frac{\sin(x/2)}{2}\right]$$

c) $u(x) = e^{-x}$ $\quad u'(x) = -e^{-x}$

$v(x) = \sin 3x$ $\quad v'(x) = 3\cos 3x$

$$f'(x) = -e^{-x} \sin 3x + 3e^{-x} \cos 3x$$

$$f'(x) = e^{-x}\left[3\cos 3x - \sin 3x\right]$$

d) $u(x) = e^x$ $\quad u'(x) = e^x$

$v(x) = \cos 4x$ $\quad v'(x) = -4\sin 4x$

$$f'(x) = e^x \cos 4x - 4e^x \sin 4x$$

$$f'(x) = e^x\left[\cos 4x - 4\sin 4x\right]$$

(e) $(\cos x + \sin x)e^{-x}$ **(f)** $e^{\sin 2x}$ **(g)** $e^{\cos x}$ **(h)** $e^{\sin x + \cos x}$

e) $u(x) = e^{-x}$ $u'(x) = -e^{-x}$

$v(x) = \cos x + \sin x$ $v'(x) = -\sin x + \cos x$

$f'(x) = -e^{-x}(\cos x + \sin x) + e^{-x}(-\sin x + \cos x)$

$f'(x) = e^{-x}\left[\cancel{-\cos x} - \sin x - \sin x \cancel{+\cos x}\right]$

$f'(x) = -2\sin x\, e^{-x}$

f) $\left(e^{\sin 2x}\right)' = e^{\sin 2x} \times (\sin 2x)'$

$\_\_\_\_ = e^{\sin 2x} \times 2\cos 2x = 2\cos 2x\, e^{\sin 2x}$

g) $\left(e^{\cos x}\right)' = e^{\cos x} \times (\cos x)' = e^{\cos x} \times (-\sin x)$

$f'(x) = -\sin x\, e^{\cos x}$

h) $\left(e^{\sin x + \cos x}\right)' = e^{\sin x + \cos x} \times (\sin x + \cos x)'$

$\_\_\_\_ = e^{\sin x + \cos x} \times (\cos x - \sin x)$