SOLVING DIFFERENTIAL EQUATIONS OF THE FORM *dy/dx = g(y)*

1 In each case, find the equation of the solution curve and then sketch its graph.

(a) $\frac{dy}{dx} = -y$, $y(0) = 1$

(b) $\frac{dy}{dx} = 2y$, $y(0) = -1$

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2 In each case, find the equation of the solution curve and then sketch its graph.

(a) $\frac{dy}{dx} = -2(y-3)$, $y(0) = 8$

(b) $\frac{dy}{dx} = -2(y(x)-8)$, $y(0) = 3$

3 Given that $\frac{dy}{dx} = \cos^2 y$ and that $y = \frac{\pi}{4}$ at $x = 0$, then which of the following is true?

A $y = \frac{1}{2}y + \frac{1}{4}\sin 2y$

B $x = \tan\left(y + \frac{\pi}{4}\right)$

C $y = \tan^{-1}(x+1)$

D $y = \tan^{-1}\left(x - \frac{\pi}{4}\right)$

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4 In Biology and Ecology, the term *desiccation* refers to the drying out (i.e. the loss of water) of the cells of a living organism. Most cells are mostly made of water. Assume that the desiccation of a cell is modelled by the solution of the following differential equation: $\frac{dV}{dt} = -kV^{\frac{2}{3}}$, where V is the volume of the water in the cell, t is time and k is an appropriate constant of proportionality.

During the intense heat of the Australian summer, the cells of a newly fallen eucalyptus leaf still contain water, but the leaf loses this water rapidly through the process of desiccation. Suppose that each leaf cell initially contains 8 μm^3 of water, but 4 hours later each cell has only 1 μm^3 of water.

(a) Find the particular solution of $\frac{dV}{dt} = -kV^{\frac{2}{3}}$, $V(0) = 8\,\mu m^3$.

(b) Find the time taken for the cells to lose all their water (assuming that the environmental conditions don't change over this time).

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5 The pressure of the atmosphere, P kilopascals (kPa), decreases according to the height h km above sea level. The rate of change of the pressure with respect to the height above sea level is proportional to the pressure at that height.

(a) Write a differential equation to describe this situation.

(b) The pressure at sea level is 101.3 kPa and it is approximately 37.3 kPa at a height of 5 km. Solve the differential equation to find P as a function of h.

(c) Estimate the air pressure at the top of Mount Everest, which is about 9 km high.

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6 In an electric circuit, a capacitor of capacitance C charged to a potential difference E is discharged through a resistance R. If q is the charge on the capacitor at time t, then $\frac{dq}{dt} = -\frac{q}{RC}$ is the differential equation describing this situation. If initially $q = EC$ then find the solution of this equation (that is, q as a function of t).

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7 Newton's law of cooling states that 'the cooling rate of a body is proportional to the difference between the temperature of the body and that of the surrounding medium.' This may be written as $\frac{dT}{dt} = -k(T - M)$, where T is the temperature at any time t and M is the temperature of the surrounding medium (a constant).

A pot of soup is cooked at 100°C. To cool the soup, it is placed in a room where the temperature is 20°C. After 20 minutes the temperature of the soup has dropped to 70°C.

(a) Find the general solution of the differential equation $\frac{dT}{dt} = -k(T - 20)$.

(b) Find the value of k.

(c) How much time will it take the pot of soup to cool to 25°C?