MEAN AND VARIANCE OF THE BINOMIAL DISTRIBUTION

1 Find the stated values for each of the following binomial distributions.

(a) $X \sim B(20, 0.7)$. Find $E(X)$, $\mathrm{Var}(X)$, $\sigma(X)$

(b) $X \sim B(100, 0.55)$. Find $E(X)$, $\mathrm{Var}(X)$, $\sigma(X)$

(c) $X \sim B\left(50, \frac{2}{3}\right)$. Find $E(X)$, $\mathrm{Var}(X)$, $\sigma(X)$

(d) $X \sim B(10, 0.5)$. Find $E(X)$, $\mathrm{Var}(X)$, $\sigma(X)$

a) $E(X) = np = 20 \times 0.7 = 14$

$\mathrm{Var}(X) = np(1-p) = 14 \times 0.3 = 4.2$

$\sigma(X) = \sqrt{\mathrm{Var}(X)} = \sqrt{4.2} = 2.05$

b) $E(X) = 100 \times 0.55 = 55$

$\mathrm{Var}(X) = 55 \times 0.45 = 24.75$

$\sigma(X) = 4.97$

c) $E(X) = 50 \times \frac{2}{3} = 33.\dot{3} = \frac{100}{3} = 33\frac{1}{3}$

$\mathrm{Var}(X) = 33.\dot{3} \times \left(\frac{1}{3}\right) = \frac{100}{9} = 11\frac{1}{9}$

$\sigma(X) = 10/3 = 3\,1/3$

d) $E(X) = 10 \times 0.5 = 5$

$\mathrm{Var}(X) = 5 \times 0.5 = 2.5 = 5/2$

$\sigma(X) = \sqrt{\frac{5}{2}} \simeq 1.58$

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2 For each of the following, find the value of the unknown.

(a) $X \sim B(15, p)$, $E(X) = 5$, $p = ?$

(b) $X \sim B(30, p)$, $E(X) = 3$, $p = ?$

$E(X) = np = 15p = 5$

so $p = \frac{1}{3}$

$E(X) = 30p = 3$

so $p = 1/10$

3 For each of the following, find the values of n and p for $X \sim B(n, p)$.

(a) $X \sim B(n, p)$, $\mu = 4.8$, $\sigma^2 = 2.88$

(b) $X \sim B(n, p)$, $\mu = 2$ and $\sigma^2 = 1.8$

a) $E(X) = np = \mu = 4.8$

$\sigma^2 = Var(X) = np(1-p) = 2.88 \quad \therefore \quad (1-p) = \frac{2.88}{4.8}$

$\therefore \quad p = 0.4 \quad$ and so $\quad n = \frac{4.8}{0.4} = 12$

b) $E(X) = np = \mu = 2$

$\sigma^2 = Var(X) = 1.8 = np(1-p)$

$\therefore \quad (1-p) = \frac{1.8}{2} \quad$ so $\quad p = 1 - \frac{1.8}{2} = 0.1$

so $n = \frac{2}{0.1} = 20$

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6 Find $P(X = 3)$ given that $E(X) = 3$ and $\mathrm{Var}(X) = 0.75$ and $X \sim B(n, p)$.

$E(X) = np = 3$

$\mathrm{Var}(X) = np(1-p) = 0.75$ so $1 - p = \dfrac{0.75}{3}$

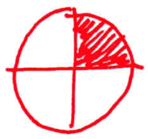
$\therefore p = 1 - \dfrac{0.75}{3} = 0.75$ and $n = \dfrac{3}{0.75} = 4$

$X \sim B(4, 0.75)$

So $p(X=3) = {}^4C_3 \; 0.75^3 \times 0.25 = 0.42187$

7 A spinner is divided into four equal sections, one of which is coloured blue. If the spinner is spun eight times, calculate the probability of obtaining less than the expected number of blue outcomes, correct to three decimal places.

$p(\text{blue}) = 1/4$ $\quad n = 8$



So $E(X) = 8 \times \dfrac{1}{4} = 2.$

So the expected number of outcomes is 2.

$P(X<2) = P(X=0) + P(X=1)$

$= {}^8C_0 \, (0.25)^0 \left(\dfrac{3}{4}\right)^8 + {}^8C_1 \left(\dfrac{1}{4}\right)^1 \left(\dfrac{3}{4}\right)^7$

$= 0.367$

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9 Consider $X \sim B(8, 0.25)$.

(a) Construct a table showing the probability distribution. Round probabilities to four decimal places.

(b) Use the rule $\mu = np$ to find $E(X)$.

(c) Use the rule $\sigma = \sqrt{\text{Var}(X)}$ to find $\sigma(X)$, correct to two decimal places.

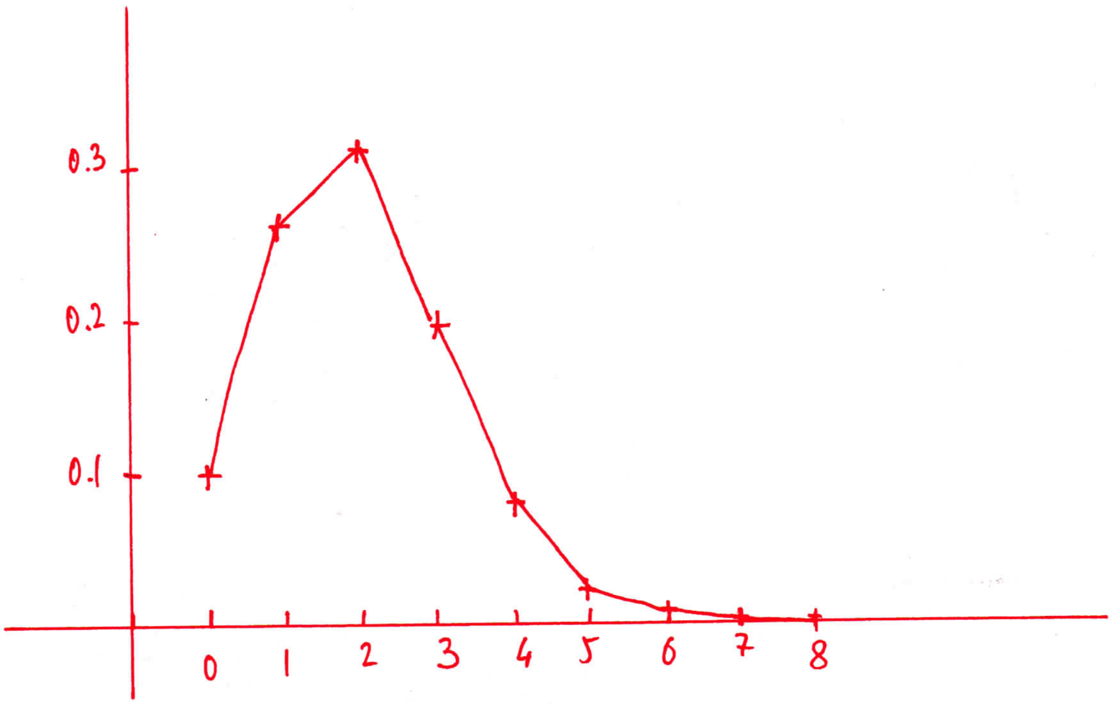
a)

x	0	1	2	3	4	5	6	7	8
$P(X=x)$	0.1001	0.2670	0.3115	0.2076	0.0865	0.0231	0.0038	0.0004	0.0000

$$P(X=i) = {}^{8}C_{i}\,(0.25)^{i}\,(0.75)^{8-i}$$

b) $\mu = np = 8 \times 0.25 = 2$

c) $\sigma = \sqrt{\text{Var}(X)} = \sqrt{2 \times (1-0.25)} = 1.22$



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10 A family has six children who are all boys or girls. Assume the probability of any child being a boy is 0.5 and that the probability is independent for each child. Find the probability of each of the following, rounding answers to four decimal places:

(a) the first two children born are male

(b) there are three boys and three girls, in any order

(c) there are more girls than boys

(d) there is at least one boy, but more girls than boys

(e) there are no consecutive births of the same sex.

a) $0.5 \times 0.5 = 0.25$

b) $P = {}^6C_3 \times (0.5)^3 \times (1-0.5)^{6-3} = {}^6C_3\,(0.5)^6 = 0.3125$

c) either 0, 1, or 2 boys. let X random variable being number of boys.

$P = P(X=0) + P(X=1) + P(X=2)$

$P = {}^6C_0 \times (0.5)^0 \times (0.5)^6 + {}^6C_1 \times (0.5)^1 (0.5)^5 + {}^6C_2\,(0.5)^2 (0.5)^4$

$P = (0.5)^6 \times \left[{}^6C_0 + {}^6C_1 + {}^6C_2\right] = 0.3438$

d) $P = P(X=1) + P(X=2)$

$P = (0.5)^6 \times \left[{}^6C_1 + {}^6C_2\right] = 0.3281$

e) so either B G B G B G or G B G B G B.

$P(\text{no consecutive births of same sex}) = \frac{2}{2^6} = \frac{1}{2^5} = 0.0313$

MEAN AND VARIANCE OF THE BINOMIAL DISTRIBUTION

11 A researcher has calculated the mean and the variance for a sample of a given random variable that has a binomial distribution.

(a) If the mean of the data set is 42 and the variance is six, determine the number of trials (n) and the probability of success (p).

(b) A new set of data is collected and the researcher notices that the mean and variance of the new set of data are double the corresponding values of the first set of data. Compare the two sets of data using appropriate calculations involving n and p.

(c) The researcher realises that the results for 10 trials have been omitted from the first set of data. When the mean is recalculated with the 10 additional results, the mean is unchanged at $\mu = 42$. What variance should the researcher expect for this set of data? Give your answer correct to two decimal places.

(d) Another researcher conducts the same experiment and records the data for 56 trials. The probability of success for this set of data is calculated to be $p = \frac{6}{7}$. What values of the mean and variance should the researcher expect to calculate for this set of data? Give your answer correct to two decimal places.

(e) Write a statement to describe the effect on the mean and variance as the number of trials increases.

a) $\mu = 42$, $Var(X) = 6$ so $\sigma = \sqrt{6}$

$np = 42$ $np(1-p) = 6$ so $p - 1 = \frac{-6}{42}$ $\therefore p = \frac{6}{7}$

$\therefore n = \frac{42}{6/7} = 49$

b) $\mu_N = 84$ $Var(N) = 12$ so $(1-p) = \frac{12}{84}$ $\therefore p = \frac{6}{7}$

$\therefore n = \frac{84}{6/7} = 98.$

c) $\mu = 42$ $n = 59$ so $p = \frac{42}{59}$

$Var(X) = 42\left(1 - \frac{42}{59}\right) = 12\frac{6}{59} = 12.10$

d) $n = 56$ $p = 6/7$

$E(X) = 56 \times \frac{6}{7} = 48$ $Var(X) = 48\left(1 - \frac{6}{7}\right) = \frac{48}{7} \simeq 6.86$

e) If the mean is the same, but the number of trials increase, p decreases which in turn, produces an increase in the number of the sample.

If the probability of success is the same but the number of trials is larger, this produces an increase in both the mean and the variance of the sample.