FURTHER EXAMPLES INVOLVING DISCRIMINANTS

1 Find the values of k for which the following quadratic equations have: **(i)** one root **(ii)** two roots.
(a) $x^2 - 3x + k = 0$ **(b)** $x^2 + kx + 3 = 0$

a) $\Delta = (-3)^2 - 4 \times k = 9 - 4k$

if $9 - 4k = 0$ (i.e, $k = 9/4$) then one root

if $9 - 4k > 0$ (i.e. $k < 9/4$) then 2 roots

b) $\Delta = k^2 - 4 \times 3 = k^2 - 12$

if $k^2 - 12 = 0$, i.e, $k = \pm\sqrt{12} = \pm 2\sqrt{3}$ then one root

if $k^2 - 12 > 0$, i.e. $k < -2\sqrt{3}$ or $k > 2\sqrt{3}$

then 2 roots.

3 For what values of m does the quadratic equation $(5m - 3)x^2 - 4mx + m + 1 = 0$ have only one root?

$\Delta = (-4m)^2 - 4 \times (5m - 3) \times (m + 1)$

$\Delta = 16m^2 - 4(5m^2 + 2m - 3)$

$\Delta = -4m^2 - 8m + 12$

The quadratic has only one root if $\Delta = 0$, i.e. we need to look for the root of Δ.

$\Delta_2 = (-8)^2 - 4 \times (-4) \times 12 = 256 = 16^2$ so 2 roots.

$m_1 = \dfrac{8 - 16}{2 \times (-4)} = 1$ or $m_2 = \dfrac{8 + 16}{2 \times (-4)} = -3$

For those values, the polynomial has only one root

FURTHER EXAMPLES INVOLVING DISCRIMINANTS

10 For what values of m does the equation $x^2 - 2mx + 8m - 15 = 0$ have: **(a)** one root **(b)** two roots?

a) $\Delta = (-2m)^2 - 4 \times 1 \times (8m - 15) = 4m^2 - 32m + 60$

when $\Delta = 0$, the equation has one root only, which is

This occurs when: $\Delta_2 = (-32)^2 - 4 \times 4 \times 60 = 64 = 8^2$

So $m_1 = \dfrac{32 - 8}{2 \times 4} = 3$ or $m_2 = 5$

b) For the equation to have 2 roots, Δ must be strictly positive, i.e. $4m^2 - 32m + 60 > 0$

which occurs when $m < 3$ or $m > 5$

(as $f(x) = 4x^2 - 32x + 60$ is a concave up parabola, therefore is positive outside of the 2 roots).