RATIONALISING DENOMINATORS

The expressions $\sqrt{a} \times \sqrt{a}$ and $\left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right)$ both have rational answers (that is, answers that do not involve surds): $\sqrt{a} \times \sqrt{a} = a$ and $\left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right) = \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2 = a - b$. For example:

- $\sqrt{3} \times \sqrt{3} = 3$
- $\left(\sqrt{7} - \sqrt{2}\right)\left(\sqrt{7} + \sqrt{2}\right) = 7 - 2 = 5$

The expansion $\left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right) = a - b$ is known as the 'difference of two squares'. You have seen this previously as $(x - y)(x + y) = x^2 - y^2$. By letting $x = \sqrt{a}$ and $y = \sqrt{b}$, we have obtained a process to convert any binomial surd into a rational number.

When you have a surd expression in the denominator of a fraction, it is normal to make the denominator into a rational number. This process is called **rationalising the denominator**.

Remember: to change a fraction without changing its value, multiply the numerator and the denominator by the same amount.

The expressions $\sqrt{a} - \sqrt{b}$ and $\sqrt{a} + \sqrt{b}$ are called **conjugate** surds, with each expression being the conjugate of the other. To convert any surd into a rational number, multiply the surd by its conjugate.

Example 20

Express with a rational denominator:

(a) $\frac{1}{\sqrt{2}}$ **(b)** $\frac{\sqrt{7}}{\sqrt{3}}$ **(c)** $\frac{1}{\sqrt{3}+\sqrt{2}}$ **(d)** $\frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}}$

Solution

(a) $\frac{1}{\sqrt{2}}$: multiply by $\frac{\sqrt{2}}{\sqrt{2}}$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

(b) $\frac{\sqrt{7}}{\sqrt{3}}$: multiply by $\frac{\sqrt{3}}{\sqrt{3}}$

$$= \frac{\sqrt{7}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{21}}{3}$$

In parts **(c)** and **(d)** the denominator is a binomial surd, so you have to multiply both numerator and denominator by the conjugate of the denominator.

(c) $\frac{1}{\sqrt{3}+\sqrt{2}} = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$$= \frac{\sqrt{3}-\sqrt{2}}{3-2}$$

$$= \sqrt{3}-\sqrt{2}$$

(d) $\frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}} = \frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}} \times \frac{2\sqrt{7}+\sqrt{3}}{2\sqrt{7}+\sqrt{3}}$

$$= \frac{2\sqrt{21}+3}{4 \times 7 - 3}$$

$$= \frac{2\sqrt{21}+3}{25}$$

Example 21

Express $\frac{\sqrt{2}}{2\sqrt{2}+1}+\frac{2}{\sqrt{3}+1}$ as a single fraction with a rational denominator.

Solution

You can add the two fractions by finding a common denominator and then rationalising the result, or by first rationalising each denominator and then adding the resulting fractions. The latter is usually the easier method, unless the denominators happen to be conjugates.

$$\begin{aligned}
\frac{\sqrt{2}}{2\sqrt{2}+1}+\frac{2}{\sqrt{3}+1} &= \frac{\sqrt{2}}{2\sqrt{2}+1}\times\frac{2\sqrt{2}-1}{2\sqrt{2}-1}+\frac{2}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1} \\
&= \frac{4-\sqrt{2}}{4\times 2-1}+\frac{2\left(\sqrt{3}-1\right)}{3-1} \\
&= \frac{4-\sqrt{2}}{7}+\frac{2\left(\sqrt{3}-1\right)}{2} \\
&= \frac{4-\sqrt{2}}{7}+\frac{\sqrt{3}-1}{1} \\
&= \frac{4-\sqrt{2}+7\left(\sqrt{3}-1\right)}{7} \\
&= \frac{7\sqrt{3}-\sqrt{2}-3}{7}
\end{aligned}$$