THE GRADIENT AS A RATE OF CHANGE

1 A trench is being dug by a team of labourers who remove V cubic metres of soil in t minutes, where $V = 10t - \frac{t^2}{20}$.

(a) State the domain of the function, i.e. the values of t during which soil is being removed.

(b) At what rate is the soil being removed at the end of 40 minutes?

(c) Are the labourers working at a constant rate?

(d) What is their initial rate of work, i.e. when $t = 0$?

(e) At what time are they removing soil at the rate of $5\,m^3$ per minute?

3 A cube of ice has an edge length of 10 cm. It melts so that its volume decreases at a constant rate and the block remains a cube. If the edge length measures 5 cm after 70 minutes, find:

(a) the rate at which the volume decreases **(b)** the volume at any time t.

4 A water tank is being emptied. The quantity Q litres of water remaining in the tank at any time t minutes after it starts to empty is given by $Q(t) = 1000(20 - t)^2$, $t \geq 0$.

(a) At what rate is the tank being emptied at any time t?

(b) How much time does it take to empty the tank? (When is $V = 0$?)

(c) At what time is the water flowing out at a rate of 20 000 litres per minute?

(d) What is the average rate at which the water flows out in the first 5 minutes?

6 A machine manufactures items at a variable rate given by $\frac{dQ}{dt} = 2t + 1$, $t \geq 0$, where Q is the number of items manufactured in a time t minutes.

At what rate is the machine working: **(a)** initially **(b)** after 10 minutes?

7 If the area of a circle is given by $A = \pi r^2$, show that the rate of change of the area with respect to the radius, $\frac{dA}{dr}$, is proportional to the radius. Find this rate when the radius is 2 cm.

8 A right circular cylinder of volume V has height h and radius of its base r. Find:

(a) the rate of change of volume with respect to height, if the radius of the base is constant

(b) the rate of change of volume with respect to the radius of the base, if the height is constant.

11 The revenue function for a particular manufacturer is $R = x\left(15 - \frac{x}{30}\right)$, where x is the number of units of the product sold. If the marginal revenue is given by $\frac{dR}{dx}$, find the marginal revenue when:

(a) $x = 6$ **(b)** $x = 15$ **(c)** $x = 225$