POLYNOMIALS - CHAPTER REVIEW

1 Perform the following polynomial divisions.

(a) $(x^3 + 2x^2 - 3x + 4) \div (x - 1)$ **(b)** $(4x^4 - 6x^2 + 10x - 40) \div (x + 3)$

2 Use the remainder theorem to find the remainder of the following.

(a) $x^3 - 4x^2 + 3x - 5$ divided by $(x - 2)$ **(b)** $x^4 + x^3 - 5x^2 + 4x - 2$ divided by $(x + 1)$

3 Use the factor theorem to find the linear factors (over the rational number field) of each polynomial.

(b) $x^3 + 7x^2 + 14x + 8$ **(c)** $x^3 + 5x^2 - x - 5$

4 Let $P(x) = (x-1)(x+2)Q(x) + ax + b$, where $Q(x)$ is a polynomial and a and b are real numbers. The polynomial $P(x)$ has a factor of $x+2$. When $P(x)$ is divided by $x-1$ the remainder is 6.

(a) Find the values of a and b.

(b) Find the remainder when $P(x)$ is divided by $(x-1)(x+2)$.

5 Let $P(x) = x^3 + ax^2 - x + 1$ be a polynomial where a is a real number. When $P(x)$ is divided by $x - 2$ the remainder is 15. Find the remainder when $P(x)$ is divided by $x + 3$.

7 The polynomial $P(x)$ is given by $P(x) = ax^3 + 15x^2 + cx - 72$, where a and c are constants. The three zeros of $P(x)$ are -3, 2 and α. Find the value of α.

8 The cubic polynomial $P(x) = x^3 + bx^2 + cx + d$ (where b, c, d are real numbers) has three real zeros: -1, α and $-\alpha$.

(a) Find the value of b. **(b)** Find the value of $c - d$.

9 The polynomial $P(x) = x^3 - 4x^2 + kx + 12$ has zeros α, β, γ.

(a) Find the value of $\alpha + \beta + \gamma$. **(b)** Find the value of $\alpha\beta\gamma$.

(c) Two of the three zeros are equal in magnitude but opposite in sign. Find the third zero and hence find the value of k.

10 Sketch graphs of each function. For what values of x is each function positive?

(a) $y=(x-1)(x+2)(x-3)$	**(b)** $y=(x-2)(x+2)^2$
(c) $y=x(x^2-1)(x+2)$	**(d)** $y=x^2(x-2)^2$