BERNOULLI TRIALS

When a coin is flipped, there are two outcomes possible: heads or tails. When a standard die is rolled, there are six outcomes possible: 1, 2, 3, 4, 5 or 6. If you are interested in whether or not you roll a particular number, like 6 for example, there are only two outcomes of interest in this scenario: 'getting a 6' and 'not getting a 6'.

The same argument can be applied to many practical situations: an archer hits the target or misses the target; a footballer scores a goal or misses; a medical test indicates the presence or absence of a disease. In all of these cases, you need to consider *success* and *failure*. When you attach a probability of p to success, then the probability of failure will be $1 - p$.

Bernoulli trials are a way of analysing situations like these where there are exactly two possible outcomes: success or failure. The trials are independent, so the outcome of one trial has no influence over the outcome of the next trial, and the number of trials is fixed. Bernoulli trials are named after Jacob Bernoulli (1654–1705).

Example 1

Decide whether each statement could represent a Bernoulli trial.

(a) You can either pass or fail an examination.
(b) You can either buy a particular brand of phone or not buy it.
(c) You can either walk, ride your bike or catch the bus to school.
(d) You can either get the job that you applied for or not get that job.

Solution

(a) Only two outcomes, so it is a Bernoulli trial.
(b) Only two outcomes, so it is a Bernoulli trial.
(c) Three possible outcomes, so it is not a Bernoulli trial.
(d) Only two outcomes, so it is a Bernoulli trial.

Bernoulli random variables

Associated with Bernoulli trials are Bernoulli random variables. Bernoulli random variables are often encoded using the convention that the number 1 is success, and 0 is failure. If X represents a Bernoulli random variable, and a probability of p is associated to success, where $0 < p < 1$, then:

$$P(X=x)=\begin{cases} p & \text{if } x=1 \\ 1-p & \text{if } x=0 \end{cases}$$

The probability distribution table for this situation is:

x	0	1
$P(X=x)$	$1-p$	p

The expected value is given by:

$$\begin{aligned} E(X) &= 0 \times (1-p) + 1 \times p \\ &= p \end{aligned}$$

The variance is given by:

$$\begin{aligned} \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= 0^2 \times (1-p) + 1^2 \times p - p^2 \\ &= p - p^2 \\ &= p(1-p) \end{aligned}$$

For a Bernoulli random variable X:

$E(X) = p$

$\text{Var}(X) = p(1-p)$