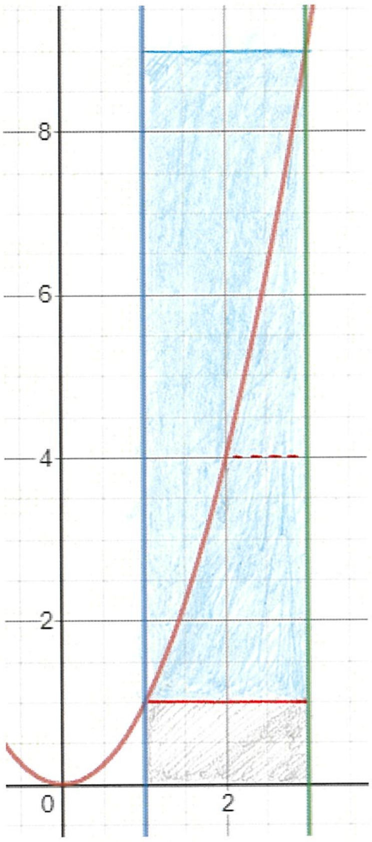
AREA UNDER A CURVE

2 Find an approximation for $\int_1^3 x^2 dx$ using rectangles with:

(a) one subinterval **(b)** two subintervals **(c)** four subintervals.



a) $2 \times 1 < \int_1^3 x^2 dx < 2 \times 9$

So $2 < \int_1^3 x^2 dx < 18$

b) two subintervals.

$1 \times 1 + 1 \times 4 < \int_1^3 x^2 dx < 1 \times 4 + 1 \times 9$

$\therefore \quad 5 < \int_1^3 x^2 dx < 13$

(which is a better approximation than at a))

c) four subintervals

$0.5 \times 1 + 0.5 \times 2.25 + 0.5 \times 4 + 0.5 \times 6.25 < \int_1^3 x^2 dx < 0.5 \times 2.25 + 0.5 \times 4 + 0.5 \times 6.25 + 0.5 \times 9$

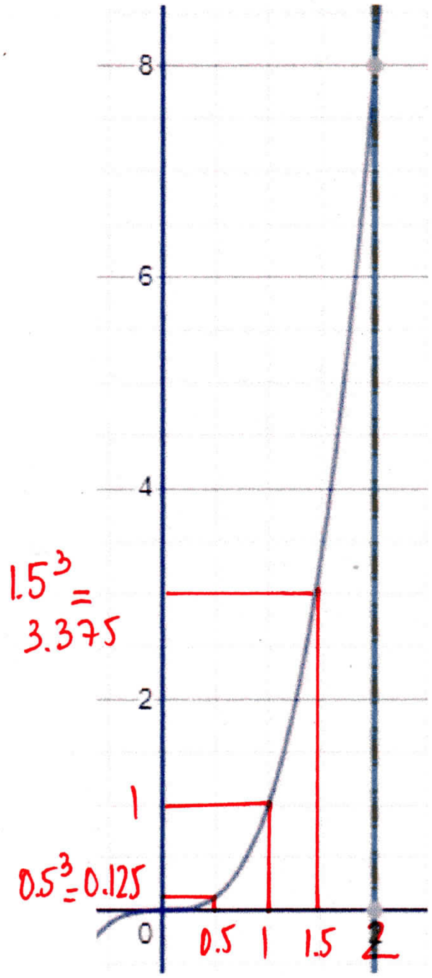
$\therefore \quad 6.75 < \int_1^3 x^2 dx < 10.75$

(which is a better approximation than at b))

AREA UNDER A CURVE

5 Find an approximation for $\int_0^2 x^3\,dx$ using rectangles with:

(a) one subinterval **(b)** two subintervals **(c)** four subintervals.



a) $0 < \text{Area} < 8 \times 2$

$0 < \text{Area} < 16$

So $0 < \int_0^2 x^3\,dx < 16$

b) $0 + 1 < \text{Area} < 1 + 1 \times 8$

$1 < \text{Area} < 9$

So $1 < \int_0^2 x^3\,dx < 9$

which is a better approximation than at a)

c) $0 + 0.5 \times 0.125 + 0.5 \times 1 + 0.5 \times 3.375 < \text{Area} < 0.5 \times 0.125 + 0.5 \times 1 + 0.5 \times 3.375 + 0.5 \times 8$

$2.25 < \text{Area} < 6.25$

So $2.25 < \int_0^2 x^3\,dx < 6.25$

which is a better approximation than at b)