SIMPLE TRIGONOMETRIC EQUATIONS

Example 20

Solve the equation $3\sin 2\theta = 1.5, 0 \le \theta \le \pi$.

Solution

$$3\sin 2\theta = 1.5$$
$$\therefore \quad \sin 2\theta = 0.5$$

$0 \le \theta \le \pi$ means that $0 \le 2\theta \le 2\pi$:

$$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Hence: $\theta = \frac{\pi}{12}, \frac{5\pi}{12}$

This solution can be checked graphically by using graphing software to find the intersection of $y = 3\sin 2\theta$ and $y = 1.5$. Over the domain $0 \le \theta \le \pi$ these functions intersect only twice. If the domain is increased, there will be two more intersections for each domain increase of π units.

Example 21

Solve the equation $\cos\left(2x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}, 0 \le x \le 2\pi$.

Solution

Let $\theta = 2x - \frac{\pi}{6}$: $\cos\theta = \frac{\sqrt{3}}{2}$

Hence: $\theta = \ldots -\frac{11\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}, 2\pi + \frac{\pi}{6}, 2\pi + \frac{11\pi}{6}, \ldots$

From the limits: $0 \le x \le 2\pi$

$$0 \le 2x \le 4\pi$$

But $2x = \theta + \frac{\pi}{6}$: $0 \le \theta + \frac{\pi}{6} \le 4\pi$

$$-\frac{\pi}{6} \le \theta \le 4\pi - \frac{\pi}{6}$$
$$\therefore \quad \theta = -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}, 2\pi + \frac{\pi}{6}, 2\pi + \frac{11\pi}{6}$$
$$\therefore \quad 2x - \frac{\pi}{6} = -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}, 2\pi + \frac{\pi}{6}, 2\pi + \frac{11\pi}{6}$$
$$2x = 0, \frac{\pi}{3}, 2\pi, \frac{7\pi}{3}, 4\pi$$
$$x = 0, \frac{\pi}{6}, \pi, \frac{7\pi}{6}, 2\pi$$

Example 22

Find the values of x for which $\cos 2x \leq \frac{1}{\sqrt{2}}$, $0 \leq x \leq 2\pi$.

Solution

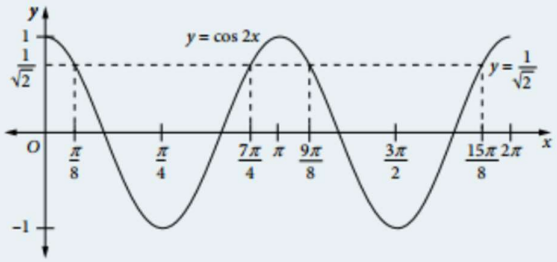
First solve the equation, then solve the inequality graphically.

Solve the equation: $\cos 2x = \frac{1}{\sqrt{2}}$ $\quad (0 \leq x \leq 2\pi)$

$$2x = \frac{\pi}{4}, \frac{7\pi}{4}, 2\pi + \frac{\pi}{4}, 2\pi + \frac{7\pi}{4} \quad \text{(as } 0 \leq 2x \leq 4\pi\text{, around the circle twice)}$$

$$x = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$$

Now sketch the graph of $y = \cos 2x$ for $0 \leq x \leq 2\pi$, showing the line $y = \frac{1}{\sqrt{2}}$ also:



Use the diagram to find where the graph of $y = \cos 2x$ is on or below the line $y = \frac{1}{\sqrt{2}}$.

Hence $\cos 2x \leq \frac{1}{\sqrt{2}}$ for $\frac{\pi}{8} \leq x \leq \frac{7\pi}{8}$ and for $\frac{9\pi}{8} \leq x \leq \frac{15\pi}{8}$.