VOLUMES OF SOLIDS OF REVOLUTION

1 Find the volume of the solid of revolution formed by rotating about the x-axis the arc of the parabola $y = x^2$ between $x = 0$ and $x = 3$.

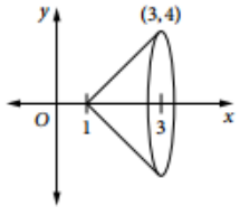
3 A cone is formed by rotating about the x-axis a segment of the line $y = 3x$ between $x = 0$ and $x = 4$. The definite integral used to calculate the volume of this solid is:

A $\int_0^4 9x^2\,dx$ **B** $\pi\int_0^4 3x^2\,dx$ **C** $\int_0^4 3x^2\,dx$ **D** $\pi\int_0^4 9x^2\,dx$

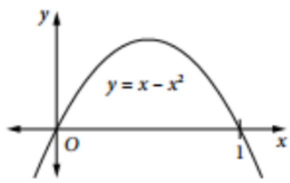
VOLUMES OF SOLIDS OF REVOLUTION

4 **(a)** Find the equation of the line passing through the points $(1, 0)$ and $(3, 4)$.

(b) A cone is formed by rotating about the x-axis the segment of the line joining the points $(1, 0)$ and $(3, 4)$. Calculate the volume of the cone.



6 The region bounded by the parabola $y = x - x^2$ and the x-axis is rotated about the x-axis. Find the volume of the solid formed.



VOLUMES OF SOLIDS OF REVOLUTION

7 Find the volume of the solid formed when the region bounded by the parabola $y = 1 - x^2$ and the x-axis is rotated about: **(a)** the x-axis **(b)** the y-axis.

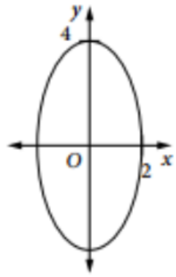
VOLUMES OF SOLIDS OF REVOLUTION

13 A hemispherical bowl of radius a units is filled with water to a depth of $\frac{a}{2}$ units. Use integration to find the volume of the water.

VOLUMES OF SOLIDS OF REVOLUTION

18 Find the volume of the solid formed when the ellipse $4x^2 + y^2 = 16$ is rotated about:

(a) the x-axis **(b)** the y-axis.



VOLUMES OF SOLIDS OF REVOLUTION

20 The region bounded by the curve $xy = 1$, the x-axis and the lines $x = 1$ and $x = a$, for $a > 1$, is rotated about the x-axis. Find V, the volume generated. Hence find $\lim_{a \to \infty} V$.

VOLUMES OF SOLIDS OF REVOLUTION

25 The area under the curve $y = e^{-x}$ between $x = 0$ and $x = 1$ is rotated about the x-axis. Find the volume of the solid of revolution.

27 Find the volume generated when the curve $y = e^{-0.5x}$, $-2 \le x \le 2$, is rotated about the x-axis.

VOLUMES OF SOLIDS OF REVOLUTION

29 **(a)** Find the area of the region bounded by the curve $y = e^{-x}$, the coordinate axes and the line $x = a$, $a > 0$.

(b) Find the limit of this area as $a \to \infty$.

(c) Find the volume of the solid generated by rotating the region in **(a)** about the x-axis and find the limit of this volume as $a \to \infty$.

VOLUMES OF SOLIDS OF REVOLUTION

31 Find the volume of the solid generated by rotating about the x-axis the area beneath the curve $y = \frac{1}{\sqrt{x-2}}$ between $x = 6$ and $x = 11$.

VOLUMES OF SOLIDS OF REVOLUTION

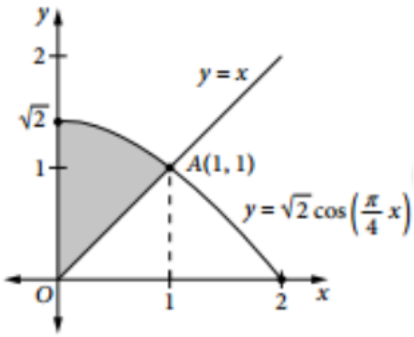
37 **(a)** Sketch the region bounded by the curves $y = 2(x^2 - 1)$ and $y = 1 - x^2$.

(b) Calculate the area of the shaded region.

(c) The region bounded by the y-axis and the curves $y = 2(x^2 - 1)$ and $y = 1 - x^2$ for $x \geq 0$, is rotated about the y-axis. Calculate the volume of the solid of revolution generated.

VOLUMES OF SOLIDS OF REVOLUTION

38 The curve $y = \sqrt{2}\cos\left(\frac{\pi}{4}x\right)$ meets the line $y = x$ at the point $A(1, 1)$, as shown in the diagram.



(a) Find the exact value of the shaded area.

(b) The shaded area is rotated about the x-axis. Calculate the volume of the solid of revolution formed.

(c) The shaded area is rotated about the y-axis. Write the integral for this volume.

(d) By using a combination of exact integration and the trapezoidal rule, as appropriate, calculate the volume of the solid in **(c)**.

VOLUMES OF SOLIDS OF REVOLUTION

42 A bowl is formed by rotating the curve $y = 8\,ln(x - 1)$ about the y-axis for $0 \leq y \leq 4$.

Calculate the capacity of the bowl, giving your answer to one decimal place.