1 In how many ways can the letters of MOTOR be arranged in a line?

4 different letters 5! permutations but 2 same letters,

so $5!/2! = 60$

2 In how many ways can the letters of NEWCASTLE be arranged in a line?

A 4! B 8! C 9! **D** $\frac{9!}{2!}$

9 letters, twice E

so $\frac{9!}{2!}$

3 The letters of PRINCIPLE are arranged in a line. In how many ways:

2P 2I 9 letters.

(a) can this be done (b) can the letters 'P' be next to each other?

So $\frac{9!}{2!2!} = 90{,}720$

8 possible choices for the location of 1st P.

then $\frac{7!}{2!}$ so $8 \times \frac{7!}{2!} = 20{,}160$

4 In how many ways can the letters of PERMUTE be arranged if:

2E 7 letters.

4 consonants, all different

(a) consonants occupy the first and the last places

(b) the vowels and the consonants occupy alternate places?

a) 4 choices for the 1st consonant, then 3 for the last 4×3 then 5 remaining letters, 2 of them are the same. So $4 \times 3 \times 5!/2! = 720$

b) 7 letters, so the 1st letter has to be a consonant. CVCVCVC

for the C: 4! for the V: $\frac{3!}{2!} = 3$ So $4! \times 3 = 72$

5 (a) In how many ways can the letters of PRECISION be arranged?

9 letters 2I 4 vowels

(b) In how many of these arrangements do the vowels occupy all the 'even' places (second, fourth, sixth, eighth)?

a) $9!/2! = 181{,}440$

b) for the C: 5! possibilities

for the V: $\frac{4!}{2!} = 12$ So $12 \times 5! = 1{,}440$

6 How many arrangements can be made of the letters of DEFINITION if:

10 letters 3I 2N

(a) the letters 'I' do not occupy the first or last place

(b) the letters 'I' are together?

a) 7 choices for 1st letter, 6 for last, then $\frac{8!}{3!2!}$ for the remaining letters,

so $7 \times 6 \times \frac{8!}{3!2!} = 141{,}120$

8 choices for the 1st I. then $\frac{7!}{2!}$ for the remaining letters.

So $8 \times \frac{7!}{2!} = 20{,}160$

7 How many arrangements of the letters of TOMATO are possible if the letters 'O' are never next to each other? Indicate whether each statement is correct or incorrect.

6 letters 2T, 2O

(a) $\frac{6!}{2! \times 2}$ (b) $\frac{5!}{2!}$ (c) $\frac{6!}{2! \times 2!} - \frac{5!}{2!}$ (d) 120

Total Number of arrangements: $\frac{6!}{2!2!}$

Arrangements with O together: 5 choices for the 1st O, then for the remaining 4 positions that $\frac{4!}{2!}$ So $5 \times \frac{4!}{2} = \frac{5!}{2!}$ So in total $\frac{6!}{2!2!} - \frac{5!}{2!}$ **C**

8 How many arrangements of the letters of PARRAMATTA are possible?

10 Letters
4A 2R 2T

$$\frac{10!}{4!\,2!\,2!} = 37,800$$

9 Seven cubes, identical except that four are red and three are black, are arranged in a row. How many different arrangements are possible?

$$\frac{7!}{4!\,3!} = 35$$

10 Three blue, three white and three red balls are placed in a row.

(a) How many different arrangements are possible?

a) $\frac{9!}{3!3!3!} = \frac{9!}{6^3} = 1,680$

(b) In how many of these arrangements are the red balls together?

b) 7 choices for the 1st red ball. Then for the 2 other colours, $\frac{6!}{3!3!}$

So $7 \times \frac{6!}{3!3!} = 140$

11 How many seven-digit numbers can be formed that contain all the digits 2, 3, 3, 3, 4, 5, 6?

3
7

$$\frac{7!}{3!} = 840$$

12 Find the number of different ways in which n students can stand in a row when two are boys, the rest are girls, and the boys all stand together.

$(n-2)$

$(n-1)$ possibilities for 1st boy. But Boy A and Boy B can be alternated, so $2(n-1)$. Then for the girls: $(n-2)!$

So $2(n-1)(n-2)! = 2(n-1)!$

13 Five drummers and five singers sit together in a row.

(a) In how many ways can this be done?

10!

(b) In how many ways can this be done if a particular drummer must not sit between two particular singers?

b) No of possibilities for the drummer to be seated between 2 particular singers

$= 8 \times 2 \times 7! = 2 \times 8!$

8 possible seats for drummer; 2 possibilities for 2 singers

So $10! - 2 \times 8!$

14 In how many ways can four people be accommodated at a hotel if there are four rooms available? (Assume that each room has enough beds for everyone.)