ZEROS OF A POLYNOMIAL

1 Given that $(z+2-i)$ is a factor, factorise $z^4+4z^3+3z^2-8z-10$ over:

(a) the set of real numbers **(b)** the set of complex numbers.

2 Solve the following for z as a complex number.

(a) $z^2-4z+8=0$ **(b)** $z^3+2z^2-2z+3=0$ **(c)** $z^6+7z^3-8=0$

3 Solve $z^3 + z^2 + 3z - 5 = 0$ for z as **(a)** a real number **(b)** a complex number.

4 Solve $z^5 + 3z^4 - z - 3 = 0$ for z as a real number.

5 What are the roots of $z^4 - 2z^3 - z + 2 = 0$ for z as a complex number?

A $1, 2$ **B** $1, 2, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ **C** $-1, -2$ **D** $-1, -2, \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

6 Find the values of the real numbers a and b such that $1 + i$ is a root of the equation $z^3 + az + b = 0$.

8 Solve $3z^3 - 4z^2 - 13z - 6 = 0$ for z if z is a real number.

9 Solve $z^4 - z^3 + 6z^2 - z + 15 = 0$ for z given that $z = 1 - 2i$ is a root of the equation.

12 Write an equation of the lowest possible degree with **(i)** complex coefficients **(ii)** rational coefficients that includes the following among its roots.

(a) $2, 1 + i$ **(b)** $\sqrt{3} + 1, 2 - i$

16 Find the real numbers k such that $z = ki$ is a root of the equation $z^3 + (2+i)z^2 + (2+2i)z + 4 = 0$. Hence, or otherwise, find the three roots of the equation.

17 Solve the following equations using a calculus method.

(a) $z^4 + 4z^3 + 5z^2 + 4z + 4 = 0$, given that it has a root of multiplicity 2.

18 If z is a complex number, solve $z^4 - 2z^2 + 9 = 0$, given that $1 + 2\sqrt{2}i = (\sqrt{2} + i)^2$.